

## Related Rates and Optimization Review #2

1. What is the maximum product of two positive numbers which are solutions to  $4x+y = 16$ ?
  
2. A box is constructed by cutting equal corners from a  $10 \times 12$  in piece of material. Find the dimensions that will maximize the volume of the box. (no top to the box)
  
3. Find the point when the line  $y=-3x+8$  that is closest to the origin.
  
4. Find the area of the largest rectangle that can be inscribed in the parabola  $y = x^2-18$  below the x-axis.
  
5. The car in the garage is leaking oil at a rate of 10 ml (or cubic cm) per hour. The oil spill is growing in a circular shape that is 0.1 cm deep. How fast is the radius changing when the circumference is  $20\pi$  cm? How much SLOWER is the radius changing when the circumference is  $30\pi$  cm?
  
6. Melted chocolate is flowing at a rate of  $3\text{ft}^3 / \text{min}$  into a giant ice cream cone with diameter 30ft and height 180ft.
  - (a) Find the volume of the melted chocolate as a function of the chocolate level  $h$ .
  - (b) How fast is the level of the cone rising when the height is 3ft?
  - (c) How fast is the radius changing when the height is 3ft?

7. A rectangle is inscribed in the parabola  $y = 16 - x^2$ . The point  $P = (x, y)$  lies on the parabola and the base is on the  $x$ -axis. The  $x$ -coordinate is moving at a rate of 3 in/second.
- How fast is the  $y$  coordinate changing when  $x = 3$ ?
  - Express the area of the rectangle as a function of  $x$  only.
  - How fast is the area changing when  $x = 3$ ?
  - How fast is the perimeter changing when  $x = 3$ ?
8. A cone with a radius of 4 feet and a height of 12 feet is filled with oil that is leaking out at the tip at a rate of 2 cu ft/hour. How fast is the height of the oil falling when the height is 3ft?  $\left( v = \frac{1}{3}\pi r^2 h \right)$ .
9. At 1:00 pm two cars are approaching an intersection. One is 340 miles away headed south at a speed of 40 mph. The other 460 miles away, is headed west at a speed of 60 miles per hour. How fast are they approaching each other at 2:00pm?
10. If a rectangle is inscribed in a right triangle with legs of 12 and 7, what is the maximum area of the rectangle?

1) 16 2)  $1.811 \times 6.379 \times 8.379$  3)  $(12/5, 4/5)$  4)  $24\sqrt{6}$  5)  $\frac{5}{\pi}; \frac{5}{3\pi}$  6) a)  $V = \frac{\pi h^3}{432}$  b)  $\frac{48}{\pi}$  ft/min c)  $\frac{4}{\pi}$  ft/min 7) a) -18 in/s b)  $A = 32x - 2x^3$  c)  $-66 \text{ in}^2/\text{s}$  d)  $-24 \text{ in/s}$  8)  $\frac{2}{\pi}$  ft/hr 9) approaching at 72 mph 10) 21 units<sup>2</sup>