

Volume of Revolution Worksheet
Disk and Washer Methods

(Integrate by hand and double check you work--also practice integrating)

$$\text{Disks: } V = \int_a^b \pi r^2 dx \text{ or } V = \int_c^d \pi r^2 dy \quad \text{Washers: } V = \int_a^b \pi (R^2 - r^2) dx \text{ or } V = \int_c^d \pi (R^2 - r^2) dy$$

$$\text{Disks: } V = \pi \int_a^b [R(x)]^2 dx \text{ or } V = \pi \int_c^d [R(y)]^2 dy \quad \text{Washers: } V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 dx \text{ or}$$

$$V = \pi \int_c^d [R(y)]^2 - [r(y)]^2 dy$$

- Find the volume of the solid of revolution generated by revolving the region bounded by $y = 6$, $y = 0$, $x = 0$, and $x = 4$ about: (a) the x -axis (**452.389**) and (b) y -axis (**301.593**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = x$, $y = 0$, and $x = 2$ about: (a) the x -axis (**8.378**) and (b) y -axis (**16.755**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{1 - x^2}$ and $y = 0$ about the x -axis. (**4.189**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = x^2$ and $y = 4$ about the x -axis. (**160.850**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = 1 - x$, $y = 0$, and $x = 0$ about: (a) the x -axis (**1.047**), (b) the y -axis (**1.047**), and (c) the line $y = -1$. (**4.189**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = x$, $y = 0$, and $2 \leq x \leq 4$ about: (a) the x -axis (**58.643**), (b) the y -axis (**117.286**), and (c) the line $x = 4$. (**33.510**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about: (a) the x -axis (**25.133**), (b) the y -axis (**80.425**), (c) the line $x = 4$ (**53.617**), and (d) the line $x = 6$. (**120.637**)
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = 2x^2$, $y = 0$, and $x = 2$ about: (a) the y -axis (**50.265**), (b) the x -axis (**80.425**), and (c) the line $y = 8$ (**187.658**),
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = x^2$ and $y = 4x - x^2$ about: (a) the x -axis (**33.510**) and (b) the line $y = 6$ (**67.021**).
- Find the volume of the solid of revolution generated by revolving the region bounded by $y = 6 - 2x - x^2$ and $y = x + 6$ about: (a) the x -axis (**152.681**) and (b) the line $y = 3$ (**67.858**).
- The region bounded by the parabola $y = 4x - x^2$ and the x -axis is revolved about the x -axis. Find the volume of the solid. (**107.233**)

Additional Practice if Needed--Disks and Washers

For problems 1 through 5, find the volume of the solid obtained by revolving about the x -axis the region with the given boundaries:

1) $f(x) = 2x + 1, y = 0, 1 \leq x \leq 4$ (**367.566**)

2) $f(x) = \sin x, y = 0, 1 \leq x \leq \pi$ (**4.935**)

3) $f(x) = \tan x, y = 0, 1 \leq x \leq \pi/4$ (**0.674**)

4) $f(x) = (1/4)x^2, g(x) = x$ (**26.808**)

5) $f(x) = \sin x, g(x) = \cos x, 0 \leq x \leq \pi/4$ (**1.571**)

For problems 6 through 8, find the volume of the solid obtained by revolving about the y -axis the region with the given boundaries:

6) $y = x^2, y = 0, 0 \leq x \leq 2$ (**25.133**)

7) $y = x^3, x = 2, y = 0$ (**40.212**)

8) $y = 1/x, y = 0, 1/4 \leq x \leq 1$ (**4.712**)

9) Find the volume of the solid obtained by rotating the region bounded by the x -axis and the graph of $y = 1 - x^2$ about the line $y = -3$. (**28.484**)

10) Find the volume of the solid obtained when the region bounded by the graphs of $y = \sqrt{x}, y = 0$, and $x = 9$ is rotated about the line $y = -2$. (**353.429**)