Name Key Per. ____ Date_____ Score ____

Section 10.3 Worksheet (Geometric Sequences)

Write the first five terms of each geometric sequence.

1) $a_1 = 24, \quad r = \frac{1}{3}$

$24, 8, \frac{8}{3}, \frac{8}{9}, \frac{8}{27}$

2) $a_n = -3a_{n-1}, \quad a_1 = -2$

$-2, 6, -12, 24, -48$

Use the formula for the general term of a geometric sequence to find the indicated term of each sequence with the given first term, a_1, and common ratio, r.

3) Find a_{12}, when $a_1 = 4, \ r = -2$.

$a_{12} = 4(-2)^{12-1}$

$a_{12} = 4(-2048)$

$a_{12} = -8192$

4) Find a_{30}, when $a_1 = -8000, \ r = \frac{1}{2}$.

$a_{30} = -8000\left(\frac{1}{2}\right)^{30-1}$

$a_{30} = \frac{-8000}{536870912} =$

$a_{30} = -\frac{125}{8,388,608}$

Write a formula for the general term (the nth term) of each geometric sequence. Then use the formula for a_n to find a_9, the 9th term of the sequence.

5) $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots$

$a_n = 5\left(-\frac{1}{5}\right)^{n-1}$

$a_9 = 5\left(-\frac{1}{5}\right)^{9-1}$

$a_9 = \frac{1}{78,125}$

6) $0.07, 0.007, 0.0007, 0.00007, \ldots$

$a_n = 0.07(0.1)^{n-1}$

$a_9 = 0.07(0.1)^{9-1}$

$a_9 = 0.07(0.00000001)$

$a_9 = 0.0000000007$

Find the sum of the finite geometric series.

7) Find the sum of the first 12 terms of the geometric sequence:

$-3, 6, -12, 24, \ldots$

$S_{12} = \frac{-3(1-(-2)^{12})}{1-(-2)}$

$S_{12} = \frac{-3(1-4096)}{3} =$

$S_{12} = -1(-4095) = 4095$

8) Find the sum of the first 11 terms of the geometric sequence:

$-\frac{1}{24}, \frac{1}{12}, -\frac{1}{6}, \frac{1}{3}, \ldots$

$S_{11} = -\frac{1}{24}\left(\frac{1-(-2)^{11}}{1-(-2)}\right)$

$S_{11} = -\frac{1}{24}\left(\frac{1+2048}{3}\right)$

$S_{11} = -\frac{683}{24}$

Find the sum of the finite geometric series.

9) $\sum_{i=1}^{6} 4^i$

$S_6 = \frac{4(1-4^6)}{1-4}$

$a_1 = 4$

$r = 4$

$n = 6$

$S_6 = 4\left(\frac{-4095}{-3}\right)$

$S_6 = 4(1365)$

$S_6 = 5460$

10) $\sum_{i=1}^{6} \left(\frac{1}{3}\right)^{i+1}$

$S_6 = \frac{1}{9}\left(\frac{1-\left(\frac{1}{3}\right)^6}{1-\frac{1}{3}}\right)$

$a_1 = \frac{1}{9}$

$r = \frac{1}{3}$

$n = 6$

$S_6 = \frac{1}{9}\left(\frac{\frac{728}{729}}{\frac{2}{3}}\right)$

$S_6 = \frac{1}{9}\left(\frac{364}{729}\right)\frac{3}{2}$

$S_6 = \frac{364}{2187}$

Find the sum of the infinite geometric series.

11) $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$

$a_1 = 1 \quad r = \frac{1}{4}$

$S_\infty = \frac{1}{1-\frac{1}{4}} =$

$S_\infty = \frac{1}{3/4} = 4/3$

12) $3-1+\frac{1}{3}-\frac{1}{9}+\ldots$

$a_1 = 3 \quad r = -\frac{1}{3}$

$S_\infty = \frac{3}{1+\frac{1}{3}} =$

$S_\infty = \frac{3}{4/3} = 9/4$

Are the following sequences arithmetic, geometric, or neither? State the sum or difference.

13) $-\frac{1}{3}, -\frac{1}{5}, -\frac{1}{7}, -\frac{1}{9}, \ldots$

neither

14) $-\frac{2}{3}, -\frac{5}{3}, -\frac{8}{3}, -\frac{11}{3}, \ldots \quad -\frac{5}{3}+\frac{2}{3} = -1$

arithmetic $d = -1$

Write a formula for the general term (the nth term) of each arithmetic sequence.

15) $a_7 = -12, \quad a_{12} = 38$

$(7, -12) \quad (12, 38)$

$d = \frac{38+12}{12-7} = \frac{50}{5} = 10$

$-12 = a_1 + (7-1)(10)$

$-12 = a_1 + 60$

$a_1 = -72$

$a_n = -72 + (n-1)10$

$a_n = -72 + 10n - 10$

$a_n = 10n - 82$

16) $a_{20} = 38, \quad a_{23} = -10$

$(20, 38) \quad (23, -10)$

$d = \frac{-10-38}{23-20} = \frac{-48}{3} = -16$

$38 = a_1 + (20-1)(-16)$

$38 = a_1 + -304$

$a_1 = 342$

$a_n = 342 + (n-1)(-16)$

$a_n = 342 + 16 - 16n$

$a_n = -16n + 358$

Find the sum.

17) $\sum_{k=1}^{10} (-3+2k)$ Arithmetic

$a_1 = -1 \quad a_{10} = 17 \quad n = 10$

$S_{10} = \frac{10}{2}(-1+17)$

$S_{10} = 5(16) = 80$

18) $\sum_{i=1}^{10} \frac{1}{3}(-2)^i$ Geometric

$a_1 = -\frac{2}{3} \quad r = -2 \quad n = 10$

$S_{10} = -\frac{2}{3}\left(\frac{1-(-2)^{10}}{1-(-2)}\right)$

$S_{10} = -\frac{2}{3}\left(\frac{-1023}{3}\right) = \frac{682}{3}$

19) $\sum_{i=1}^{4} \frac{(i-4)}{i!}$ neither

$a_1 = -3 \quad a_2 = -1 \quad a_3 = -\frac{1}{6} \quad a_4 = 0$

$-3 + -1 + -\frac{1}{6} = -4\frac{1}{6}$

20) $\sum_{j=1}^{\infty} \frac{1}{2}(-3)^j$ Infinite Geometric

$a_1 = -\frac{3}{2} \quad r = -3$

NO sum because $|-3| > 1$