11.2
Finding Limits Using the Properties of Limits

- Find the limit by graphing
- Finding the limit using properties
- Simplifying to find the limit
- Finding one-sided limits of a piecewise function

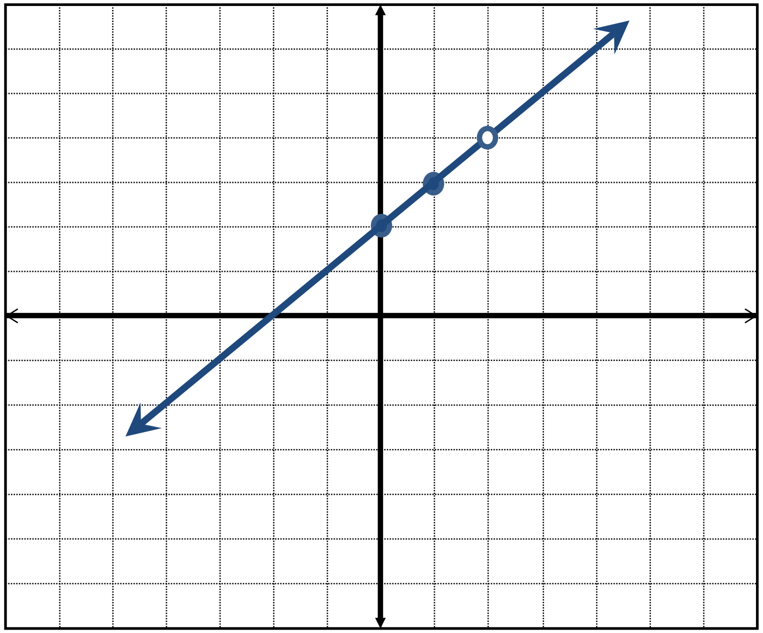
Graphing a Function to Find the Limit

1. $f(x) = \dfrac{x^2 - 4}{x - 2}, \quad \lim_{x \to 2} f(x)$

$$f(x) = \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$f(x) = x + 2, \quad x \neq 2$$

$$\lim_{x \to 2} x + 2 = \boxed{4}$$



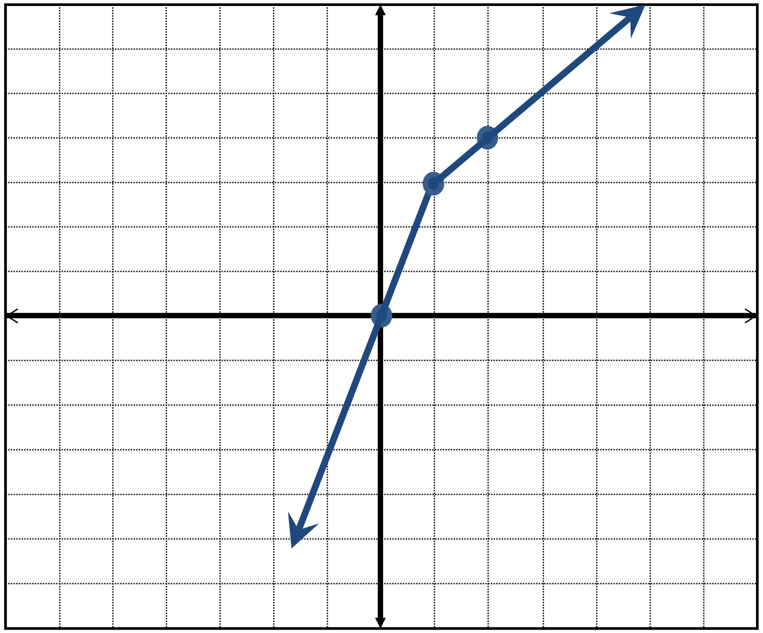
Graphing a Function to Find the Limit

2. $f(x) = \begin{cases} 3x & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}, \quad \lim_{x \to 1} f(x)$

$$\lim_{x \to 1^-} 3x = 3$$

$$\lim_{x \to 1^+} x + 2 = 3$$

$$\lim_{x \to 1} f(x) = \boxed{3}$$



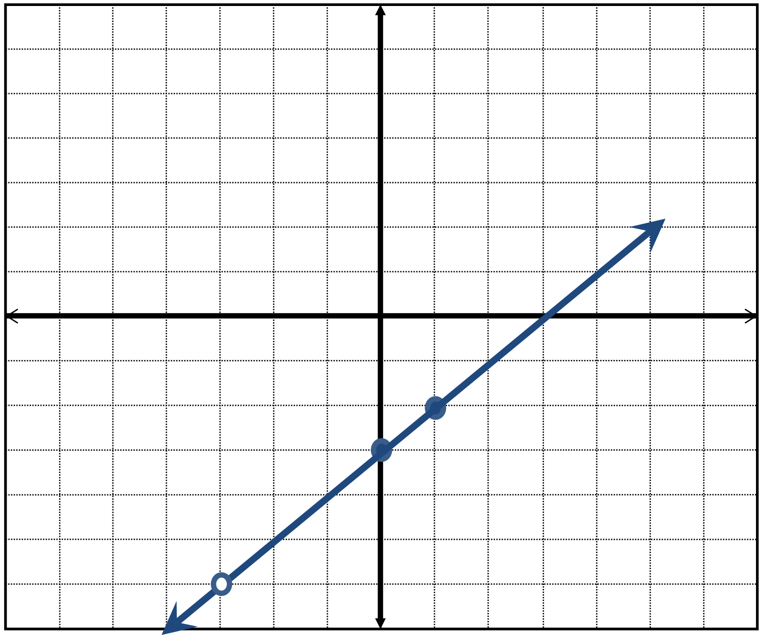
Graphing a Function to Find the Limit

3. $f\left(x\right)=\dfrac{x^2-9}{x+3}, \quad \lim_{x\to -3} f\left(x\right)$

$$f\left(x\right)=\frac{\left(x+3\right)\left(x-3\right)}{x+3}$$

$$f\left(x\right)=x-3, \quad x\neq -3$$

$$\lim_{x\to -3} x-3=\boxed{-6}$$



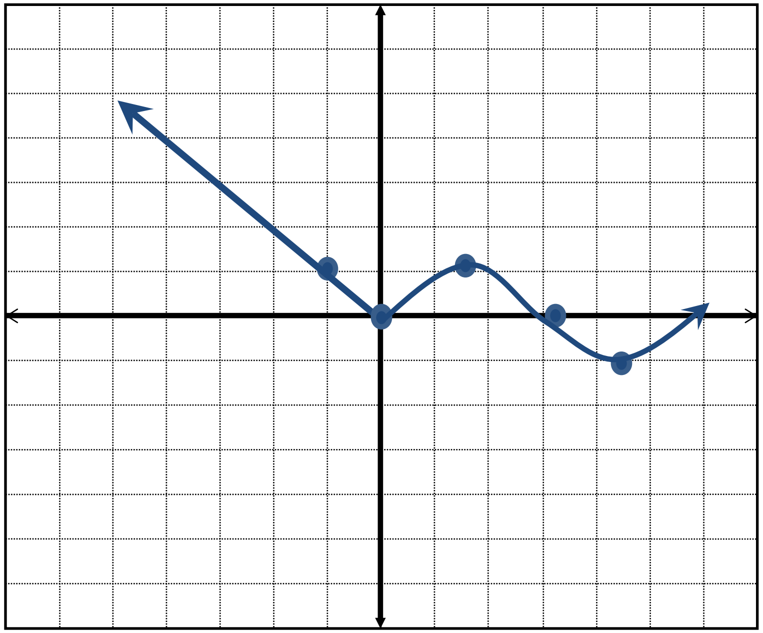
Graphing a Function to Find the Limit

4. $f(x)=\begin{cases} -x & \text{if } x<0 \\ \sin x & \text{if } x\geq 0 \end{cases}, \quad \lim_{x\to 0} f(x)$

$$\lim_{x\to 0^-}(-x)=0$$

$$\lim_{x\to 0^+}\sin x = 0$$

$$\lim_{x\to 0} f(x)=\boxed{0}$$



Using Limit Properties Find the Limit

5. $\lim_{x \to 2} 5x^2$

$\lim_{x \to 2} 5(2)^2$

$\lim_{x \to 2} 5(4) = \boxed{20}$

6. $\lim_{x \to 2} \frac{3x}{x-4}$

$\lim_{x \to 2} \frac{3(2)}{2-4}$

$\lim_{x \to 2} \frac{6}{-2} = \boxed{-3}$

Using Limit Properties Find the Limit

7. $$\lim_{x \to -1} \sqrt{5x^2 + 4}$$

$$\lim_{x \to -1} \sqrt{5(-1)^2 + 4}$$

$$\lim_{x \to -1} \sqrt{9} = \boxed{3}$$

8. $$\lim_{x \to \frac{\pi}{6}} \frac{\cos x}{1 + \sin x}$$

$$\lim_{x \to \frac{\pi}{6}} \frac{\cos \frac{\pi}{6}}{1 + \sin \frac{\pi}{6}}$$

$$\lim_{x \to \frac{\pi}{6}} \frac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}}$$

$$\lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3}}{2} \div \frac{3}{2} = \boxed{\frac{\sqrt{3}}{3}}$$

Finding Limits When Denominator is Zero

9. $$\lim_{x\to 3}\frac{x^2-x-6}{x^2-9} = \frac{0}{0}$$

$$\lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)(x+3)}$$

Factoring

$$\lim_{x\to 3}\frac{x+2}{x+3}$$

$$\lim_{x\to 3}\frac{3+2}{3+3} = \boxed{\frac{5}{6}}$$

Finding Limits When Denominator is Zero

10. $$\lim_{x \to 1} \frac{x^2 - 1}{x^3 - 1} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{(x+1)(x-1)}{(x-1)(x^2+x+1)}$$

Factoring

$$\lim_{x \to 1} \frac{x+1}{x^2+x+1}$$

$$\lim_{x \to 1} \frac{1+1}{1^2+1+1} = \boxed{\frac{2}{3}}$$

Finding Limits When Denominator is Zero

11. $$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\frac{4}{4(x+4)} - \frac{1(x+4)}{4(x+4)}}{x}$$

Simplifying a Complex Rational Expression

$$\lim_{x\to 0} \frac{-x}{4(x+4)} \bullet \frac{1}{x}$$

$$\lim_{x\to 0} \frac{-1}{4(x+4)}$$

$$\lim_{x\to 0} \frac{-1}{4(0+4)} = \boxed{-\frac{1}{16}}$$

Finding Limits When Denominator is Zero

12. $$\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \frac{0}{0}$$

$$\lim_{x \to 9} \frac{\left(\sqrt{x}-3\right)\left(\sqrt{x}+3\right)}{\left(x-9\right)\left(\sqrt{x}+3\right)}$$

Multiplying by a Conjugate

$$\lim_{x \to 9} \frac{\cancel{x-9}}{\cancel{\left(x-9\right)}\left(\sqrt{x}+3\right)}$$

$$\lim_{x \to 9} \frac{1}{\sqrt{x}+3}$$

$$\lim_{x \to 9} \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

Finding One-Sided Limits

13. $f(x) = \begin{cases} x^2 + 6 & \text{if } x < 2 \\ x^3 + 2 & \text{if } x \geq 2 \end{cases}$

a) $\lim_{x \to 2^-} =$

$\lim_{x \to 2^-} (2)^2 + 6 =$

$\boxed{10}$

b) $\lim_{x \to 2^+} =$

$\lim_{x \to 2^+} (2)^3 + 2 =$

$\boxed{10}$

c) $\lim_{x \to 2} =$

$\boxed{10}$

Finding One-Sided Limits

14. $f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & \text{if } x \neq 4 \\ 7 & \text{if } x = 4 \end{cases}$

a) $\lim_{x \to 4^-} \left(\dfrac{x^2 - 16}{x - 4} \right) =$

$$\lim_{x \to 4^-} \frac{(x+4)(x-4)}{x-4} =$$

$$\lim_{x \to 4^-} x + 4 =$$

$$\lim_{x \to 4^-} 4 + 4 =$$

$\boxed{8}$

b) $\lim_{x \to 4^+} \left(\dfrac{x^2 - 16}{x - 4} \right) =$

$$\lim_{x \to 4^+} \frac{(x+4)(x-4)}{x-4} =$$

$$\lim_{x \to 4^+} x + 4 =$$

$$\lim_{x \to 4^+} 4 + 4 =$$

$\boxed{8}$

c) $\lim_{x \to 4} =$

$\boxed{8}$