

Limits and Continuity (11.3)

Continuity

A function f is continuous at point a if:

I. $f(a)$ exists

II. $\lim_{x \rightarrow a} f(x)$ exists

III. $\lim_{x \rightarrow a} f(x) = f(a)$

Determining Whether a Functions is Continuous at a Number: Use the definition of continuity to determine whether f is continuous at a .

1) $f(x) = \frac{x+7}{x-7}$

a. $a = 7$

b. $a = -7$

2) $f(x) = \begin{cases} x-4 & \text{if } x \leq 4 \\ x^2 + x - 4 & \text{if } x > 4 \end{cases}$

a. $a = 0$

b. $a = 4$

3) $f(x) = \frac{x^2 + 8x}{x^2 - 8x}$

a. $a = 0$

b. $a = 8$

4) $f(x) = \begin{cases} 2-x & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$

a. $a = 1$

b. $a = 7$

Determining Where a Function is Discontinuous: Determine for what numbers, if any, the given function is discontinuous.

$$5) f(x) = \frac{x+2}{x^2-3x-10}$$

$$6) f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

$$7) f(x) = \frac{x-5}{x^2-5x}$$

$$8) f(x) = \begin{cases} x-2 & \text{if } x \leq 2 \\ x^2-1 & \text{if } x > 2 \end{cases}$$

$$9) f(x) = \frac{1-\cos x}{x}$$

$$10) f(x) = \begin{cases} 7x & \text{if } x < 6 \\ 41 & \text{if } x = 6 \\ x^2+6 & \text{if } x > 6 \end{cases}$$

Find the constant c that makes $f(x)$ continuous on $(-\infty, \infty)$.

$$11) f(x) = \begin{cases} cx+1 & \text{if } x \leq 3 \\ cx^2-1 & \text{if } x > 3 \end{cases}$$