11.3
Limits and Continuity

- Determine whether a function is continuous at a number
- Determine where a functions is discontinuous.

Determining Continuity

1. $f(x)=\dfrac{x+7}{x-7}$

a. $a=7$

I. $f(7)=\dfrac{7+7}{7-7}$

$f(7)=\dfrac{14}{0}=\textit{undefined}$

II. $\lim\limits_{x\to 7}=\dfrac{7+7}{7-7}$

$\lim\limits_{x\to 7}=\dfrac{14}{0}=DNE$

III. $\lim\limits_{x\to 7} f(x)\neq f(7)$

Discontinuous

b. $a=-7$

I. $f(-7)=\dfrac{-7+7}{-7-7}$

$f(-7)=\dfrac{0}{-14}=0$

II. $\lim\limits_{x\to -7}=\dfrac{-7+7}{-7-7}$

$\lim\limits_{x\to -7}=\dfrac{0}{-14}=0$

III. $\lim\limits_{x\to -7} f(x)= f(-7)$

Continuous

Determining Continuity

2. $f(x)=\begin{cases} x-4 & \text{if } x \le 4 \\ x^2+x-4 & \text{if } x > 4 \end{cases}$

a. $a=0$

I. $f(0)=0-4=-4$

II. $\lim\limits_{x\to 0} = 0-4=-4$

III. $\lim\limits_{x\to 0} f(x) = f(0)$

Continuous

b. $a=4$

I. $f(4)=4-4=0$

II. $\lim\limits_{x\to 4^-} = 4-4=0$

$\lim\limits_{x\to 4^+} = 4^2+4-4=16$

$\lim\limits_{x\to 4} = DNE$

III. $\lim\limits_{x\to 4} f(x) \neq f(4)$

Discontinuous

Determining Continuity

3. $f(x)=\dfrac{x^2+8x}{x^2-8x}$

a. $a=0$

Discontinuous

b. $a=8$

Discontinuous

4. $(x)=\begin{cases}2-x & \text{if } x<1\\ 1 & \text{if } x=1\\ x^2 & \text{if } x>1\end{cases}$

a. $a=1$

Continuous

b. $a=7$

Continuous

Determining Discontinuity

5. $f(x) = \dfrac{x+2}{x^2-3x-10}$

$$f(x) = \frac{x+2}{(x-5)(x+2)}$$

$$x \neq 5, -2$$

f is discontinuous at $x = -2$ and 5

f is continuous at all values except $x = -2$ and 5

6. $$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

I. $f(3) = 6$

II. $\lim\limits_{x \to 3} = 3 + 3 = 6$

III. $\lim\limits_{x \to 3} f(x) = f(3)$

f is continuous at all values of x

Determining Discontinuity

7. $f(x)=\dfrac{x-5}{x^2-5x}$

$$f(x)=\frac{x-5}{x(x-5)}$$

$$x\neq 0,5$$

f is discontinuous at $x=0$ and 5

8. $f(x)=\begin{cases} x-2 & \text{if } x\leq 2 \\ x^2-1 & \text{if } x>2 \end{cases}$

I. $f(2)=0$

II. $\lim\limits_{x\to 2^-} = 2-2=0$

$\lim\limits_{x\to 2^+} = 2^2-1=3$

$\lim\limits_{x\to 2} = DNE$

III. $\lim\limits_{x\to 2} f(x) = f(2)$

f is discontinuous at $x=2$

Determining Discontinuity

9. $f(x)=\dfrac{1-\cos x}{x}$

f is discontinuous at $x=0$

10. $f(x)=\begin{cases} 7x & \text{if } x<6 \\ 41 & \text{if } x=6 \\ x^2+6 & \text{if } x>6 \end{cases}$

I. $f(6)=41$

II. $\lim\limits_{x\to 6^-}=7(6)=42$

$\lim\limits_{x\to 6^+}=6^2+6=42$

$\lim\limits_{x\to 6}=42$

III. $\lim\limits_{x\to 6} f(x)\neq f(6)$

f is continuous at all values except $x=6$