3.11
Related Rates

1. Air is being pumped into a spherical balloon at a rate of 5 cm^3/min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

$$V = \frac{4}{3}\pi r^3$$

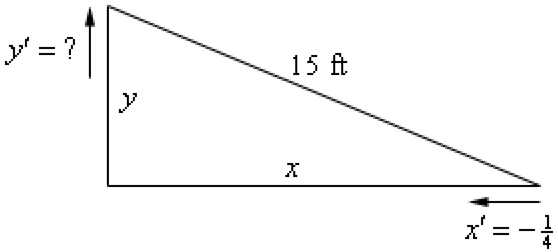
$$\frac{dV}{dt} = 5 \quad \mathrm{cm}^3/\mathrm{min} \qquad\qquad r = 10 \quad cm$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi (10)^2 \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{1}{80\pi} \approx 0.004 \ \ cm/\mathrm{min}}$$

2. A 15 foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of ¼ ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?



$$x^2 + y^2 = 15^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x = 10 - 3 = 7$$

$$\frac{dx}{dt} = -\frac{1}{4}$$

$$7^2 + y^2 = 15^2$$

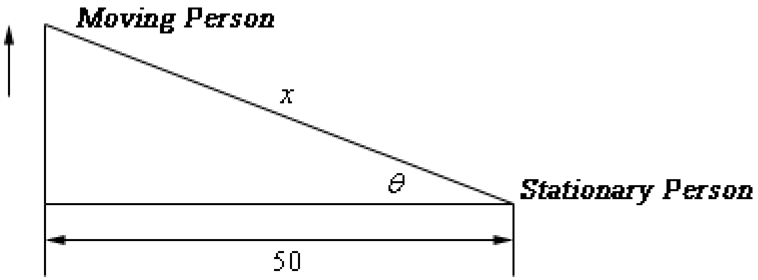
$$y = 4\sqrt{11}$$

$$2(7)\left(-\frac{1}{4}\right) + 2\left(4\sqrt{11}\right)\frac{dy}{dt} = 0$$

$$\frac{7}{2} = 8\sqrt{11}\frac{dy}{dt}$$

$$\boxed{\frac{dy}{dt} = \frac{7}{16\sqrt{11}} \approx 0.13 \quad ft/\text{sec}}$$

3. Two people are 50 feet apart. One of them starts walking north at a rate so that the angle shown in the diagram below is changing at a constant rate of 0.01 rad/min. At what rate is distance between the two people changing when radians?



$$\cos\theta = \frac{50}{x}$$

$$x = 50\sec\theta$$

$$\frac{d\theta}{dt} = 0.01$$

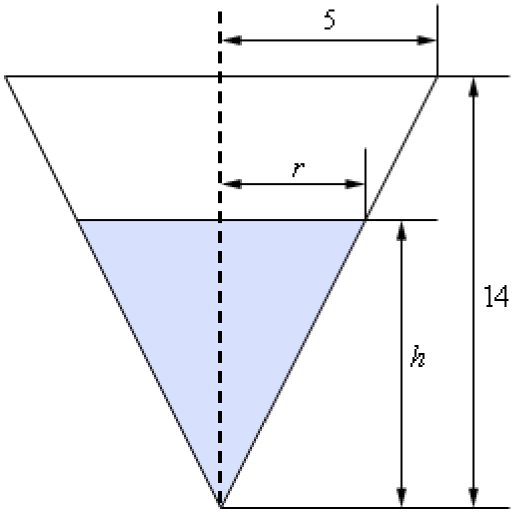
$$\theta = 0.5$$

$$\frac{dx}{dt} = 50\sec\theta\tan\theta\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = 50\sec(0.5)\tan(0.5)(0.01)$$

$$\frac{dx}{dt} \approx 0.311 \quad ft/\min$$

4a) A tank of water in the shape of a cone is leaking water at a constant rate of 2 ft³/hr. The base radius of the tank is 5 ft and the height of the tank is 14 ft. At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?



$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3}\left(\frac{5}{14}h\right)^2 h$$

$$V = \frac{\pi}{3}\left(\frac{25}{196}\right)h^3$$

$$\frac{dV}{dt} = \pi\left(\frac{25}{196}\right)h^2 \frac{dh}{dt}$$

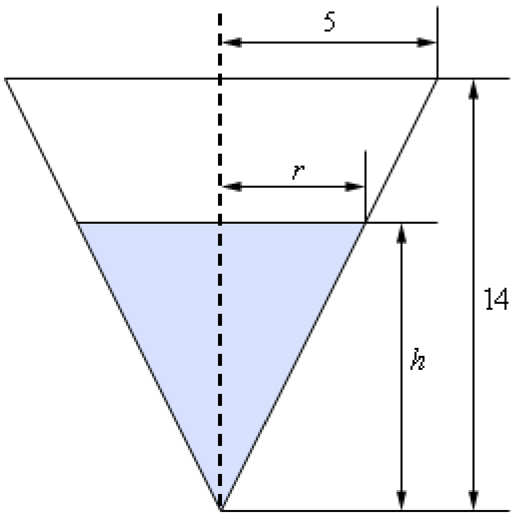
$$-2 = \pi\left(\frac{25}{196}\right)(36)\frac{dh}{dt}$$

$$\frac{r}{5} = \frac{h}{14}$$

$$r = \frac{5}{14}h \quad \text{or} \quad h = \frac{14}{5}r$$

$$\boxed{\frac{dh}{dt} = -\frac{98}{225\pi} \approx -0.139 \quad ft/hr}$$

4b) A tank of water in the shape of a cone is leaking water at a constant rate of . The base radius of the tank is 5 ft and the height of the tank is 14 ft. At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 ft?



$$r = \frac{5}{14} h$$

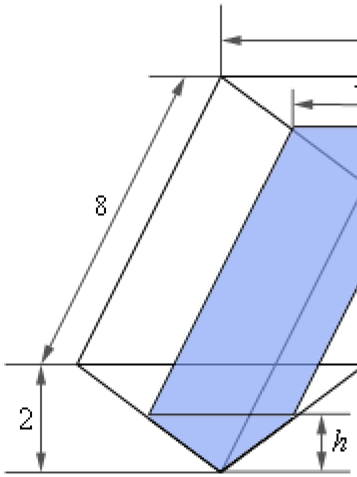
$$\frac{dr}{dt} = \frac{5}{14} \frac{dh}{dt}$$

$$\frac{dr}{dt} = \frac{5}{14}(-0.139)$$

$$\frac{dh}{dt} \approx -0.139 \quad ft/h$$

$$\frac{dr}{dt} \approx -0.05 \quad ft/h$$

5. A trough of water is 8 meters deep and its ends are in the shape of isosceles triangles whose width is 5 meters and height is 2 meters. If water is being pumped in at a constant rate of . At what rate is the height of the water changing when the water has a height of 120 cm?



$$V = \frac{1}{2}whL$$

$$V = \frac{1}{2}\left(\frac{5}{2}h\right)h(8)$$

$$V = 10h^2$$

$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$

$$\frac{w}{5} = \frac{h}{2}$$

$$h = \frac{2}{5}w \quad \text{or} \quad w = \frac{5}{2}h$$

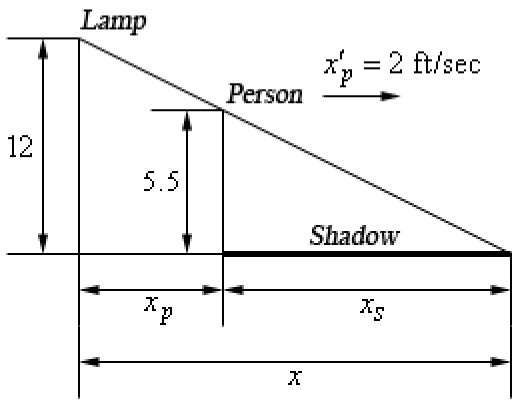
$$\frac{dV}{dt} = 6$$

$$h = 120cm = 1.2m$$

$$6 = 20(1.2)\frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.25 \ m/\sec$$

6a). A light is on the top of a 12 ft tall pole and a 5ft 6in tall person is walking away from the pole at a rate of 2 ft/sec. At what rate is the tip of the shadow moving away from the pole when the person is 25 ft from the pole?



$$x = x_p + x_s$$

$$x = x_p + \frac{11}{13}x_p$$

$$x = \frac{24}{13}x_p$$

$$\frac{dx}{dt} = \frac{24}{13}\frac{dx_p}{dt}$$

$$\frac{dx}{dt} = \frac{24}{13}(2)$$

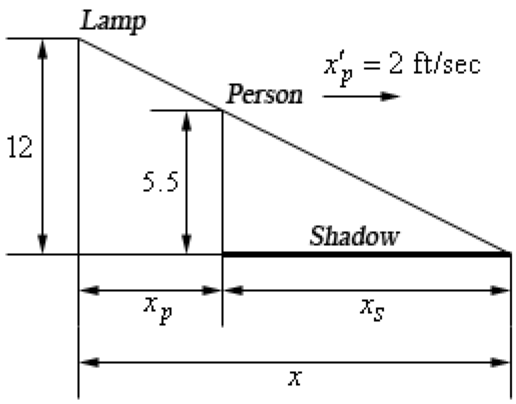
$$\frac{x_s}{x_p + x_s} = \frac{5.5}{12}$$

$$x_p = \frac{13}{11}x_s \quad \text{or} \quad x_s = \frac{11}{13}x_p$$

$$\frac{dx_p}{dt} = 2 \qquad x_p = 25$$

$$\boxed{\frac{dx}{dt} = \frac{48}{13} \approx 3.69\ ft/\text{sec}}$$

6b). A light is on the top of a 12 ft tall pole and a 5ft 6in tall person is walking away from the pole at a rate of 2 ft/sec. At what rate is the tip of the shadow moving away from the person when the person is 25 ft from the pole?



$$x = x_p + x_s$$

$$\frac{dx}{dt} = \frac{dx_p}{dt} + \frac{dx_s}{dt}$$

$$\frac{dx_p}{dt} = 2$$

$$3.69 = 2 + \frac{dx_s}{dt}$$

$$\frac{dx}{dt} = \frac{48}{13} \approx 3.69\ ft/\sec$$

$$\frac{dx_s}{dt} \approx 1.69\ ft/\sec$$