Notes 47 Name ________________________

Product and Quotient Rules of Derivatives (3.3)

Finding the Derivative of Common Functions: Differentiate the function.

1. $f(x) = \sin x$

$f' = \cos x$

2. $f(x) = \cos x$

$f' = -\sin x$

3. $f(x) = \ln x$

$f' = \frac{1}{x}$

4. $f(x) = 5^x$

$f' = 5^x \ln 5$

Product Rule

$f = u \cdot v \longrightarrow f' = u'v + v'u$

Finding the Derivative Using the Product Rule: Differentiate the function.

5. $f(x) = x^2 \ln x$

$f' = 2x \ln x + x^2\left(\frac{1}{x}\right)$

$f' = 2x \ln x + x$

$\boxed{f' = x(2\ln x + 1)}$

6. $f(x) = \frac{1}{2}x^3 + 4x^2 \cos x$

$f' = \frac{3}{2}x^2 + 8x\cos x + 4x^2(-\sin x)$

$f' = \frac{3}{2}x^2 + 8x\cos x - 4x^2 \sin x$

$\boxed{f' = x\left(\frac{3}{2}x + 8\cos x - 4x\sin x\right)}$

Quotient Rule

$$f = \frac{u}{v} \longrightarrow f' = \frac{u'v - v'u}{v^2}$$

Finding the Derivative Using the Quotient Rule: Differentiate the function.

7. $f(x) = \dfrac{3x+2}{x^4 - 5x}$

$$f' = \frac{3(x^4-5x) - (3x+2)(4x^3-5)}{(x^4-5x)^2}$$

$$f' = \frac{3x^4 - 15x - 12x^4 - 8x^3 + 15x + 10}{(x^4-5x)^2}$$

$$\boxed{f' = \frac{-9x^4 - 8x^3 + 10}{(x^4-5x)^2}}$$

8. $f(x) = \dfrac{3e^x}{7\cos x - \sqrt[3]{x}}$

$$f' = \frac{3e^x(7\cos x - x^{1/3}) - 3e^x(-7\sin x - \frac{1}{3}x^{-2/3})}{(7\cos x - x^{1/3})^2}$$

$$\boxed{f' = \frac{21e^x\cos x - 3e^x x^{1/3} + 21e^x\sin x + e^x x^{-2/3}}{(7\cos x - x^{1/3})^2}}$$

Finding a Tangent Line: Find an equation of the tangent line to the curve at the given point. Verify your answer by graphing the function and tangent line to the right.

9. $f(x) = \dfrac{x^2 - x - 2}{x+3}$ at $(-1, 0)$

$$f' = \frac{(2x-1)(x+3) - (x^2-x-2)}{(x+3)^2}$$

$$f' = \frac{2x^2 + 6x - x - 3 - x^2 + x + 2}{(x+3)^2}$$

$$f' = \frac{x^2 + 6x - 1}{(x+3)^2} = f'(-1) = -\frac{3}{2}$$

$$\boxed{y = -\frac{3}{2}x - \frac{3}{2}}$$