Notes 51 **Name** _____

Derivatives of Logarithmic Functions (3.9)

Derivatives of Exponential Functions: Differentiate.

1) $y = x^3 4^{2x}$

$dy = (3x^2 4^{2x} + x^3 4^{2x} \ln 4 (2))\,dx$

$\frac{dy}{dx} = x^2 4^{2x} (3 + 2x \ln 4)$

2) $f(x) = 5^{3x^2}$

$f'(x) = 5^{3x^2} \ln 5 (6x)\,dx$

$f'(x) = 6x 5^{3x^2} \ln 5\,dx$

Derivatives of Logarithmic Functions: Differentiate.

3) $y = \log_4 \cos x$

$y = \frac{\ln \cos x}{\ln 4}$

$y = \frac{1}{\ln 4} \ln \cos x$

$dy = \frac{1}{\ln 4} \cdot \frac{1}{\cos x} \cdot -\sin x\,dx$

$\frac{dy}{dx} = \frac{-\tan x}{\ln 4}$

4) $f(x) = \log_2 (1 + 4x^{-1}), \quad x = 4$

$f(x) = \frac{\ln (1 + 4x^{-1})}{\ln 2}$

$f'(x) = \frac{1}{(1 + 4x^{-1}) \ln 2} (-4x^{-2})\,dx$

$f'(4) = \frac{1}{2 \ln 2} \left(-\frac{1}{4}\right) = -\frac{1}{8 \ln 2}$

Logarithmic Differentiation: Use logarithmic differentiation to find the derivative of the function.

5) $y = \frac{x^2 \sqrt[3]{7x - 14}}{(1 + x^2)^4}$

$\ln y = \ln \frac{x^2 (7x - 14)^{1/3}}{(1 + x^2)^4}$

$\ln y = 2 \ln x + \frac{1}{3} \ln (7x - 14) - 4 \ln (1 + x^2)$

$\frac{dy}{y} = \frac{2}{x} dx + \frac{7}{3(7x - 14)} dx - \frac{8x}{1 + x^2} dx$

$\frac{dy}{dx} = \left(\frac{2}{x} + \frac{7}{3(7x - 14)} - \frac{8x}{1 + x^2}\right) y$

$\frac{dy}{dx} = \left(\frac{2}{x} + \frac{7}{3(7x - 14)} - \frac{8x}{1 + x^2}\right) \left(\frac{x^2 \sqrt[3]{7x - 14}}{(1 + x^2)^4}\right)$

6) $y = \frac{(x^2 - 8)^{1/3} \sqrt{x^3 + 1}}{x^6 - 7x + 5}$

$\ln y = \frac{1}{3} \ln (x^2 - 8) + \frac{1}{2} \ln (x^3 + 1) - \ln (x^6 - 7x + 5)$

$\frac{dy}{y} = \left(\frac{2x}{3(x^2 - 8)} + \frac{3x^2}{2(x^3 + 1)} - \frac{6x^5 - 7}{x^6 - 7x + 5}\right) dx$

$\frac{dy}{dx} = \left(\frac{2x}{3(x^2 - 8)} + \frac{3x^2}{2(x^3 + 1)} - \frac{6x^5 - 7}{x^6 - 7x + 5}\right) y$