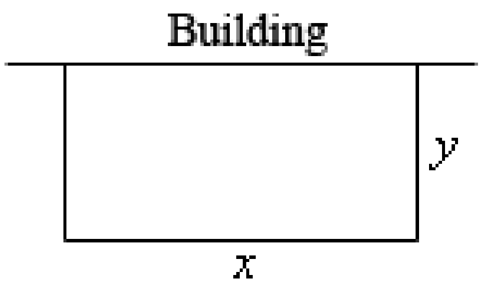
4.7
Optimization

1. We need to enclose a field with a fence. We have 500 feet of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.



$$A = xy \qquad 500 = x + 2y$$

$$x = 500 - 2y$$

$$A = y(500 - 2y)$$

$$A = 500y - 2y^2$$

$$dA = (500 - 4y)\,dy$$

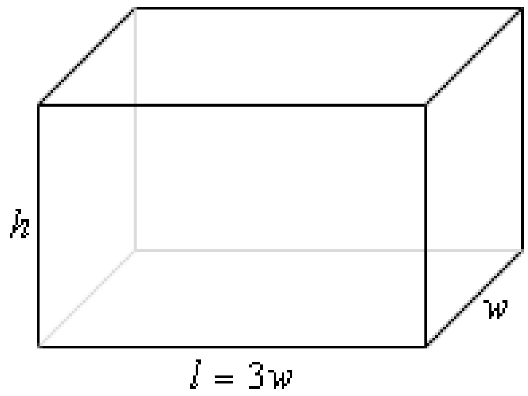
$$0 = 500 - 4y$$

$y = 125$ Critical Value (Rel. Max.)

$$\boxed{125\,ft \;\; X \;\; 250\,ft}$$

$$A = 31,250 \text{ ft}^2$$

2. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10/ft^2 and the material used to build the sides cost \$6/ft^2. If the box must have a volume of 50ft^3 determine the dimensions that will minimize the cost to build the box.



$$50 = 3w^2 h$$

$$LA_C = 6(8w)h$$

$$BA_C = 10(2)(3w^2)$$

$$C = 60w^2 + 48wh = 60w^2 + 48w\left(\frac{50}{3w^2}\right)$$

$$C = 60w^2 + \frac{800}{w}$$

$$C' = 120w - \frac{800}{w^2}$$

$$0 = 120w^3 - 800$$

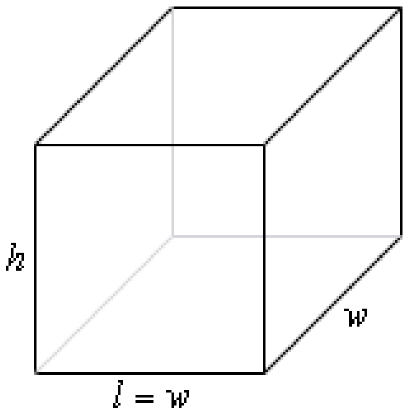
$$w = 1.88$$

$$l = 3w = 5.64$$

$$h = \frac{50}{3(1.88)^2} = 4.7$$

$$1.88\,ft,\ 5.64\,ft,\ 4.7ft$$

3. We want to construct a box with a square base and we only have 10 m² of material to use in construction of the box. Assuming that all the material is used in the construction process determine the maximum volume that the box can have.



$$V = w^2 h$$

$$10 = 2w^2 + 4wh$$

$$h = \frac{5 - w^2}{2w}$$

$$V = w^2\left(\frac{5 - w^2}{2w}\right)$$

$$V = \frac{5}{2}w - \frac{w^3}{2}$$

$$V' = \frac{5}{2} - \frac{3}{2}w^2$$

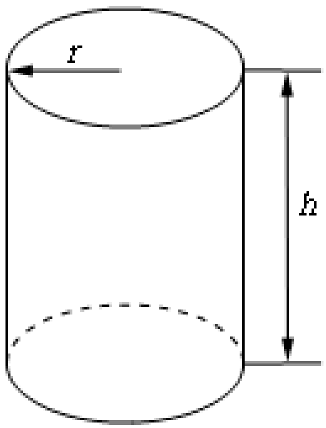
$$0 = 5 - 3w^2$$

$$w \approx 1.29$$

$$h \approx 1.29$$

$$V = (1.29)^2(1.29) \approx \boxed{2.15 \text{ m}^3}$$

4. A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction.



$$A = 2\pi rh + 2\pi r^2$$

$$1500 = \pi r^2 h$$

$$h = \frac{1500}{\pi r^2}$$

$$A = 2\pi r\left(\frac{1500}{\pi r^2}\right) + 2\pi r^2$$

$$A = \frac{3000}{r} + 2\pi r^2$$

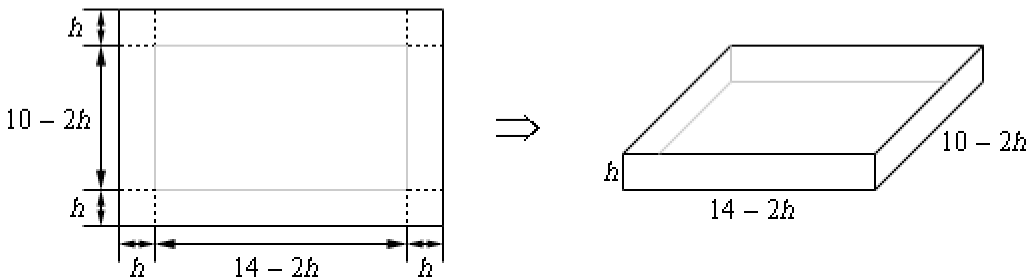
$$A' = -\frac{3000}{r^2} + 4\pi r$$

$$0 = -3000 + 4\pi r^3$$

$$r \approx 6.2 \text{ cm}$$

$$h \approx 12.4 \text{ cm}$$

5. We have a piece of cardboard that is 14 inches by 10 inches and we're going to cut out square corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.



$$V = h(14-2h)(10-2h)$$

$$V = 4h^3 - 48h^2 + 140h$$

$$V' = 12h^2 - 96h + 140$$

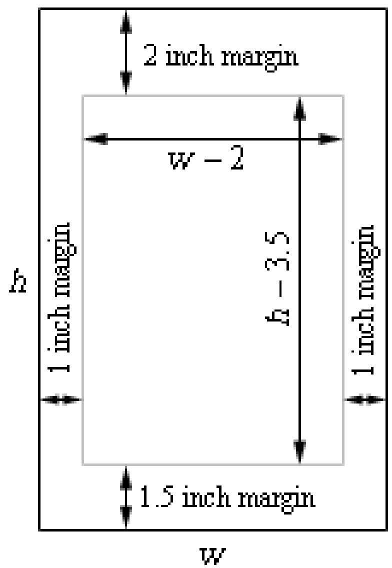
$$0 = 3h^2 - 24h + 35$$

$$h = \frac{24 \pm \sqrt{156}}{6}$$

$$h \approx \cancel{6.1,}\ 1.9$$

$$\boxed{h \approx 1.9\ in}$$

5. A printer needs to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions will give the largest printed area?



$$200 = wh$$

$$h = \frac{200}{w}$$

$$A = (w-2)(h-3.5)$$

$$A = (w-2)\left(\frac{200}{w} - 3.5\right)$$

$$A = 207 - 3.5w - \frac{400}{w}$$

$$A' = -3.5 + \frac{400}{w^2}$$

$$0 = -3.5w^2 + 400$$

$$w \approx 10.7 \text{ in.}$$

$$h \approx 18.7 \text{ in.}$$