

5.1.5.2 (Riemann Sums/The Definite Integral)

Riemann Sum	$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x dx$ $\Delta x = \frac{b-a}{n}; \quad n = \text{the number of subintervals}$
Fundamental Theorem of Calculus (Part 1)	$F(x) = \int_a^b f(x) dx = F(b) - F(a)$

Divide the interval $[a, b]$ into $n = 3$ subintervals of equal length and compute the sum of the areas (a) using the right hand sum, (b) the left hand sum, c) the midpoint rule, d) ~~the trapezoidal rule~~ and e) the integral definition.

$$1) \int_0^6 (x^2 - 4) dx \quad \Delta x = \frac{6-0}{3} = 2$$

$$R_3 = 2(f(2) + f(4) + f(6))$$

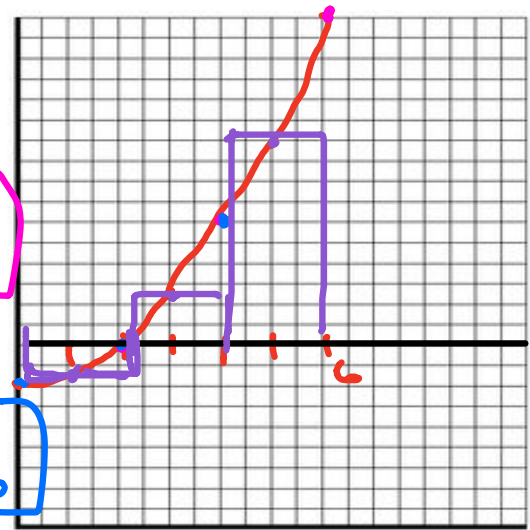
$$R_3 = 2(0 + 12 + 32) = \boxed{88}$$

$$L_3 = 2(f(0) + f(2) + f(4))$$

$$L_3 = 2(-4 + 0 + 12) = \boxed{16}$$

$$M_3 = 2(f(1) + f(3) + f(5)) =$$

$$M_3 = 2(-3 + 5 + 21) = \boxed{46}$$



$$\int_0^6 (x^2 - 4) dx$$

$$\left. \frac{x^3}{3} - 4x + C \right|_0^6$$

$$(72 - 24 + C) - (0 - 0 + C) = \boxed{48}$$

Divide the interval $[a,b]$ into $n = 3$ subintervals of equal length and compute the sum of the areas (a) using the right hand sum, (b) the left hand sum, c) the midpoint rule, d) the trapezoidal rule and e) the integral definition.

$$2) \int_1^3 (-x^3 + 4x - 2) dx$$

$$\Delta x = \frac{3-1}{3} = \frac{2}{3}$$

$$R_3 = \frac{2}{3} \left(f\left(\frac{5}{3}\right) + f\left(\frac{7}{3}\right) + f(3) \right) =$$

$$L_3 = \frac{2}{3} \left(f(1) + f\left(\frac{4}{3}\right) + f\left(\frac{7}{3}\right) \right) =$$

$$M_3 = \frac{2}{3} \left(f\left(\frac{4}{3}\right) + f(2) + f\left(\frac{8}{3}\right) \right) =$$

