

(5.2 Extension)
Applications of the Definite
Integral

Evaluate

$$\int_0^1 f(x)dx = 6 \quad \int_0^2 f(x)dx = 4 \quad \int_2^5 f(x)dx = -2$$

1. $\int_0^5 f(x)dx =$

2. $\int_1^2 f(x)dx =$

$$\int_0^2 f(x)dx + \int_2^5 f(x)dx =$$

$$\int_0^2 f(x)dx - \int_0^1 f(x)dx =$$

$$4 - 2 = \boxed{2}$$

$$4 - 6 = \boxed{-2}$$

Evaluate

$$\int_0^1 f(x)dx = 6 \quad \int_0^2 f(x)dx = 4 \quad \int_2^5 f(x)dx = -2$$

3. $\int_1^5 f(x)dx =$

4. $\int_0^0 f(x)dx =$

$$\int_2^5 f(x)dx + \int_1^2 f(x)dx =$$

$$\boxed{0}$$

$$-2 - 2 = \boxed{-4}$$

Evaluate

$$\int_0^1 f(x)dx = 6 \quad \int_0^2 f(x)dx = 4 \quad \int_2^5 f(x)dx = -2$$

5. $\int_2^0 f(x)dx =$

$$-\int_0^2 f(x)dx =$$

$$\boxed{-4}$$

6. $\int_5^1 f(x)dx =$

$$-\int_1^5 f(x)dx =$$

$$\boxed{4}$$

Evaluate the Integral and the Total Area

7. $f(x) = x^2 + 2$ over $[1, 2]$

$$a, b) \int_1^2 (x^2 + 2) dx =$$
$$\frac{x^3}{3} + 2x \Big|_1^2$$

$$\left(\frac{8}{3} + 4 \right) - \left(\frac{1}{3} + 2 \right) =$$

$$\frac{13}{3}$$

Evaluate the Integral and the Total Area

8. $f(x) = x^2 - 4$ over $[-1, 3]$

a) $\int_{-1}^3 (x^2 - 4) dx =$

$$\left. \frac{x^3}{3} - 4x \right|_{-1}^3$$

$$(9 - 12) - \left(-\frac{1}{3} + 4 \right) =$$

$$\boxed{-\frac{20}{3}}$$

b) $\int_{-1}^0 (x^2 - 4) dx + \int_0^3 (x^2 - 4) dx$

$$\left. \frac{x^3}{3} - 4x \right|_{-1}^0 + \left. \frac{x^3}{3} - 4x \right|_0^3$$

$$\left(-\frac{1}{3} + 4 \right) - \left(\frac{8}{3} - 8 \right) + (9 - 12) - \left(\frac{8}{3} - 8 \right) =$$

$$\boxed{\frac{34}{3}}$$

Evaluate the Integral and the Total Area

9. $f(x) = -2x^2 - 6x + 8$ over $[-1, 4]$

a) $\int_{-1}^4 (-2x^2 - 6x + 8)dx =$ b) $\int_{-1}^1 (-2x^2 - 6x + 8)dx + \int_4^1 (-2x^2 - 6x + 8)dx =$

$$\frac{-2x^3}{3} - 3x^2 + 8x \Big|_{-1}^4 \qquad \frac{-2x^3}{3} - 3x^2 + 8x \Big|_{-1}^1 + \frac{-2x^3}{3} - 3x^2 + 8x \Big|_4^1$$

$$\frac{193}{3}$$

$$\frac{617}{3}$$

Evaluate the Integral and the Total Area

10. $f(x) = 2x - \frac{1}{3}x^2$ over $[0, 9]$

a) $\int_0^9 (2x - \frac{1}{3}x^2) dx =$

$$x^2 - \frac{x^3}{9} \Big|_0^9$$

0

b) $\int_0^6 (2x - \frac{1}{3}x^2) dx + \int_6^9 (2x - \frac{1}{3}x^2) dx =$

$$x^2 - \frac{x^3}{9} \Big|_0^6 + x^2 - \frac{x^3}{9} \Big|_6^9$$

24

11. Evaluate $\int_{-5}^4 f(x)dx$, if $f(x) = \begin{cases} -4 & , x \leq -3 \\ 2 & , -3 < x < 0 \\ 1 & , x \geq 0 \end{cases}$

$$\int_{-5}^{-3} -4dx + \int_{-3}^0 2dx + \int_0^4 1dx =$$

$$-4x \Big|_{-5}^{-3} + 2x \Big|_{-3}^0 + x \Big|_0^4$$

$$(12 - 20) + (0 + 6) + (4 - 0) =$$

2

$$12. \int_{-5}^5 f(x) dx$$

$$\int_{-5}^{-2} f(x) dx + \int_{-2}^4 f(x) dx + \int_4^5 f(x) dx =$$

$$-4.5 + 8 - 0.5 =$$

3

