Area Between Curves (6.1)

Area Between Curves

1. $y = x^2$ and $y = \sqrt{x}$

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x\left(x^3 - 1\right) = 0$$

$$x = 0, 1$$

$$(0,0), (1,1)$$

$$\int_0^1 \left(\sqrt{x} - x^2\right) dx$$

$$\frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big|_0^1$$

$$\left(\frac{2}{3} - \frac{1}{3}\right) - (0) = \boxed{\frac{1}{3}}$$

Area Between Curves

2. $y = 2x^2 + 10$ and $y = 4x + 16$

$$2x^2 + 10 = 4x + 16$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x = 3, -1$$

$$(3, 28), (-1, 12)$$

$$\int_{-1}^{3} \left[(4x + 16) - (2x^2 + 10) \right] dx$$

$$\int_{-1}^{3} \left(-2x^2 + 4x + 6 \right) dx$$

$$-\frac{2}{3}x^3 + 2x^2 + 6x \Big|_{-1}^{3}$$

$$(18) - \left(-\frac{10}{3} \right) = \boxed{\frac{64}{3}}$$

Area Between Curves

3. $y = \sin x,\ y = \cos x,\ x = \dfrac{\pi}{2}$ and the $y-$axis.

$\sin x = \cos x$

$x = \dfrac{\pi}{4}$

$\left(\dfrac{\pi}{4}, \dfrac{\sqrt{2}}{2}\right)$

$$\int_0^{\pi/4} \left(\cos x - \sin x\right)dx + \int_{\pi/4}^{\pi/2} \left(\sin x - \cos x\right)dx$$

$$\left(\sin x + \cos x\right)\Big|_0^{\pi/4} + \left(-\cos x - \sin x\right)\Big|_{\pi/4}^{\pi/2}$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0+1) + (-1) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$2\sqrt{2} - 2 \approx 0.8$$

Area Between Curves

4. $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$

$$\frac{1}{2}y^2 - 3 = y + 1$$

$$\frac{1}{2}y^2 - y - 4 = 0$$

$$y^2 - 2y - 8 = 0$$

$$y = 4, -2$$

$$(5, 4), (-1, -2)$$

$$\int_{-2}^{4} \left[(y+1) - \left(\frac{1}{2}y^2 - 3 \right) \right] dy$$

$$\int_{-2}^{4} \left(-\frac{1}{2}y^2 + y + 4 \right) dy$$

$$-\frac{1}{6}y^3 + \frac{y^2}{2} + 4y \Big|_{-2}^{4}$$

$$\left(-\frac{32}{3} + 8 + 16 \right) - \left(\frac{4}{3} + 2 - 8 \right)$$

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