Volumes Using Disks and Washers (6.3)

Volumes Using Disks and Washers

1. $y = x^2 - 4x + 5,\ x = 1,\ x = 4$ and the x-axis about the x-axis.

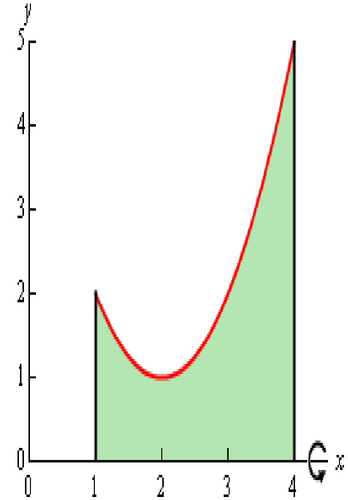
$$V = \int_1^4 \pi\left[\left(x^2 - 4x + 5\right)^2 - (0)^2\right]dx$$

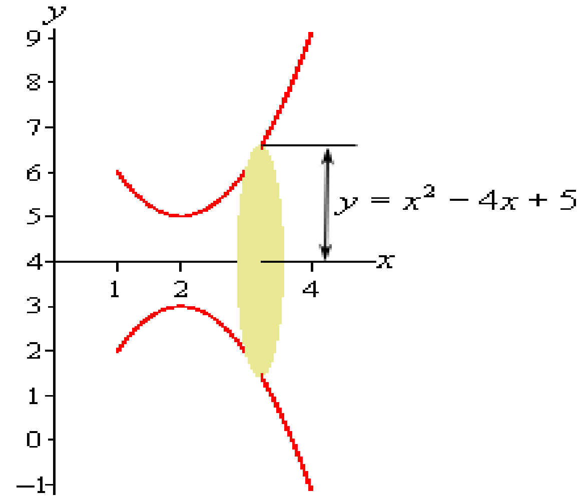
$$V = \pi\int_1^4 \left[\left(x^2 - 4x + 5\right)^2\right]dx$$

$$V = \pi\int_1^4 \left(x^4 - 8x^3 + 26x^2 - 40x + 25\right)dx$$

$$V = \pi\left(\frac{x^5}{5} - 2x^4 + \frac{26x^3}{3} - 20x^2 + 25x\right)\Bigg|_1^4$$

$$V = \boxed{\frac{78}{5}\pi}$$





Volumes Using Disks and Washers

2. $y = \sqrt[3]{x}$ and $y = \dfrac{x}{4}$ that lies in the first quadrant about the y-axis.

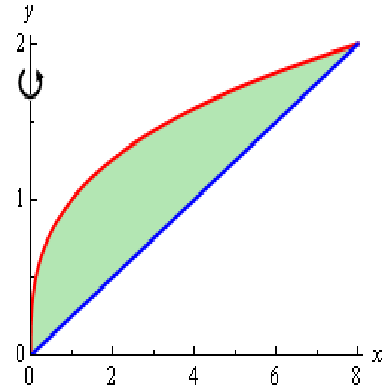
$$x = y^3, \quad x = 4y$$

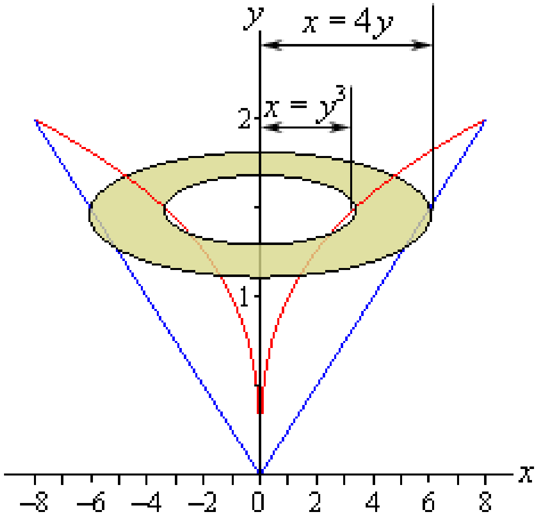
$$V = \int_0^2 \pi\left[(4y)^2 - \left(y^3\right)^2\right]dy$$

$$V = \pi\int_0^2 \left(16y^2 - y^6\right)dy$$

$$V = \left(\frac{16y^3}{3} - \frac{y^7}{7}\right)\pi\Bigg|_0^2$$

$$V = \boxed{\frac{512}{21}\pi}$$





Volumes Using Disks and Washers

3. $y = x^2 - 2x$ and $y = x$ about the line $y = 4$

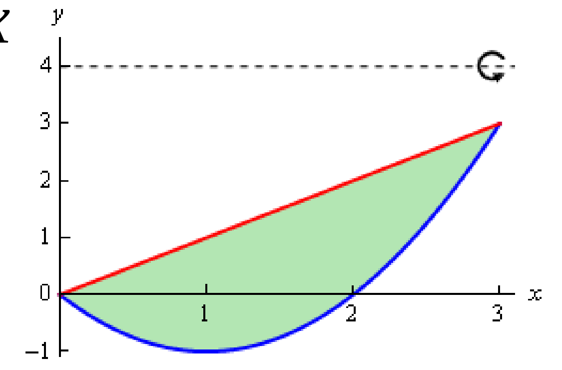
$$V = \int_0^3 \pi\left[\left(4-\left(x^2-2x\right)\right)^2 - \left(4-\left(x\right)\right)^2\right]dx$$

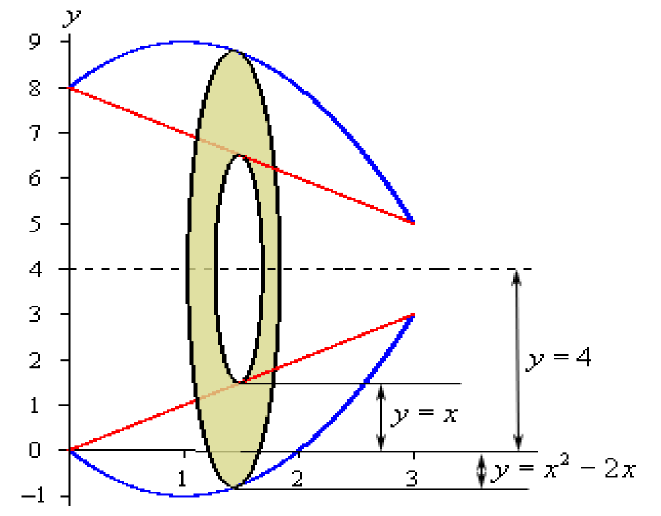
$$V = \pi\int_0^3 \left[\left(4-x^2+2x\right)^2 - \left(4-x\right)^2\right]dx$$

$$V = \int_0^3 \pi\left(x^4 - 4x^3 - 5x^2 + 24x\right)dx$$

$$V = \pi\left(\frac{x^5}{5} - x^4 - \frac{5x^3}{3} + 12x^2\right)\Bigg|_0^3$$

$$V = \boxed{\frac{153\pi}{5}}$$

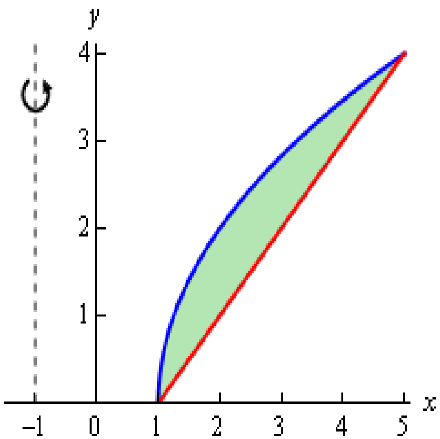




Volumes Using Disks and Washers

4. $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$.

$$V = \int_0^4 \pi\left[\left(y+1-(-1)\right)^2 - \left(\frac{y^2}{4}+1-(-1)\right)^2\right]dy$$



$$V = \pi\int_0^4 \left[\left(y+2\right)^2 - \left(\frac{y^2}{4}+2\right)^2\right]dy$$

$$V = \pi\int_0^4 \left(4y - \frac{y^4}{16}\right)dy$$

$$V = \pi\left(2y^2 - \frac{y^5}{80}\right)\Bigg|_0^4$$

$$V = \boxed{\frac{96\pi}{5}}$$