Name _____ Period _____ Score _____

HW74: Cumulative Half-Test Review #1

Find the indicated limit.

1. $\lim_{x\to 5}\dfrac{\dfrac{1}{x-2}-\dfrac{1}{3}}{x-5}$

2. $\lim_{x\to 1}\dfrac{\sqrt{37-x}-6}{x-1}$

3. $\lim_{x\to\infty}\dfrac{3x-2}{5x^2+4x+1}$

4. $\lim_{x\to-\infty}\dfrac{2x^2+1}{3x-5}$

Determine for what numbers, if any, the given function is discontinuous.
Use the definition of continuity to show the discontinuity.

5. $f(x)=\dfrac{x+1}{x^2-3x-4}$

6. $f(x)=\begin{cases}1-x & \text{if } x<1\\ 2 & \text{if } x=1\\ x^2+3 & \text{if } x>1\end{cases}$

Differentiate.

7. $y=\dfrac{x^2}{2x+1}$

8. $y=-5\sin 3x\sec 2x$

Differentiate using implicit differentiation.

9. $5x^2y-2x+y^2=7$

10. $\sin x+\sin y=xy^2$

Differentiate using logarithmic differentiation.

11. $y=\dfrac{5\sqrt{x^2-2x}}{x^4-7x^2+9}$

12. $y=\dfrac{\sqrt[3]{x^2}\left(3x^2-4x\right)}{(x^4-2)^2}$

Find the equation of the tangent line at the given value or point.

13. $y=2\tan x$ at $x=\dfrac{\pi}{4}$

14. $x^3-y^3=7$ at $(-1,-2)$

Find the intervals on which f(x) is increasing, decreasing, concave up, and concave down.

15. $f(x)=\dfrac{x}{x-4}$

16. $f(x)=2x^3-6x+5$

Find the values of c that satisfies the Mean Value Theorem.

17. $f(x)=x^2-5x+4$ on $[0,2]$

18. $f(x)=\sqrt{x-1}$ on $[1,5]$

Find the absolute minimum and maximum on the given interval.

19. $f(x) = x^4 - 2x^2 + 4$ on $[-2,1]$

20. $f(x) = -\cos 2x, \quad \left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

(Calculator is necessary). Solve each related rate problem. Round answers to the nearest thousandths.

21. At noon a distress call is made from Ship A to ship B for medical supplies. Ship A is 200 km west of Ship B. Ship A is sailing South at 35 km/h and ship B is sailing north at 25 km/h. Ship A sends a boat to Ship B at 2:00pm. How fast is the distance between the ships changing at this time?

22. Air is being pumped into a spherical balloon at a rate of 7 cubic centimeters per second. What is the rate of change of the radius at the instant the volume equals 36π?

(Calculator is necessary). Solve each optimization problem.

23. A baseball park has 3200ft of fencing and wants to fence off a rectangular section for batting cages that sit next to a building. They won't need fence along the building. What are the dimensions of the batting cages that would maximize the area?

24. A water tank has the shape of an inverted right circular cone with height 12 ft and base radius of 6 ft. If water is being pumped into the tank at a rate of 1.5 ft^3/min. Estimate the rate at which the water level is rising when the water is 3 ft deep.

Integrate.

25. $\int_0^2 xe^{x^2}\,dx$

26. $\int \frac{y}{\sqrt{y^2+1}}\,dy$

27. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 2}\,dx$

28. $\int_{-3}^{1} |x+2|\,dx$

Find the following estimations using a Riemann Sum.

29. $y = x^2 - 1$ on $[-1,2]$ when $n = 3$ using right hand endpoints.

30. $y = -x^2 + 2$ on $[-3,5]$ when $n = 4$ using left hand endpoints.

Find the area between the curve and the x-axis of the following function.

31. $y = -\sin x$ from $\left[0, \frac{3\pi}{2}\right]$.

Answers:

1) $-\frac{1}{9}$

2) $-\frac{1}{12}$

3) 0

4) $-\infty$

5) Continuous at all values of x except 4 and -1

6) Continuous at all values of x except 1

7) $\frac{2x(x+1)}{(2x+1)^2}$

8) $-5\sec 2x\left(3\cos 3x+2\sin 3x\tan 2x\right)$

9) $\frac{dy}{dx}=\frac{2-10xy}{5x^2+2y}$

10) $\frac{dy}{dx}=\frac{y^2-\cos x}{\cos y-2xy}$

11) $\frac{dy}{dx}=\left(\frac{x-1}{x^2-2x}-\frac{4x^3-14x}{x^4-7x^2+9}\right)\frac{5\sqrt{x^2-2x}}{x^4-7x^2+9}$

12) $\frac{dy}{dx}=\left(\frac{2}{3x}+\frac{6x-4}{3x^2-4x}-\frac{8x^3}{x^4-2}\right)\frac{\sqrt[3]{x^2}\left(3x^2-4x\right)}{(x^4-2)^2}$

13) $y=4\left(x-\frac{\pi}{4}\right)+2$

14) $y=\frac{1}{4}(x+1)-2$

15) $Dec:(-\infty,2)\cup(2,\infty)\ CU:(2,\infty)\ CD:(-\infty,2)$

16) $Inc:(-\infty,-1)\cup(1,\infty)\ \ Dec:(-1,1)\ CU:(0,\infty)\ CD:(-\infty,0)$

17) $c=1$

18) $c=2$

19) $Max:\ f(-2)=12\ Min:\ f(-1)=3\ \&\ f(1)=3$

20) $Max:\ f(0)=0\ \&\ f\left(\frac{\pi}{2}\right)=0\ Min:\ f\left(-\frac{\pi}{4}\right)=-2\ \&\ f\left(\frac{3\pi}{4}\right)=-2$

21) $30.870\ km/h$

22) $0.062\ cm^3/h$

23) $800\text{x}600=128,000\ units^2$

24) $0.212\ ft/\min$

25) $\frac{e^2-1}{2}$

26) $\sqrt{y^2+1}+C$

27) $\ln 3-\ln 2$

28) 5

29) 2
30) −24
31) 3