

**HW79: Cumulative Half-Test Review #2****Find the indicated limit.**

1. 
$$\lim_{x \rightarrow -7} \frac{x+7}{\frac{1}{x+4} + \frac{1}{3}}$$

2. 
$$\lim_{x \rightarrow 3} \frac{\sqrt{28-x}-5}{3-x}$$

3. 
$$\lim_{x \rightarrow -\infty} \frac{7x^2-2}{5x^2-3x-2}$$

4. 
$$\lim_{x \rightarrow \infty} \frac{7x^2+x+1}{4x-5}$$

**Determine for what numbers, if any, the given function is discontinuous.****Use the definition of continuity to show the discontinuity.**

5. 
$$f(x) = \frac{x+4}{2x^2+9x+4}$$

6. 
$$f(x) = \begin{cases} 3-x^2 & \text{if } x < 1 \\ x+2 & \text{if } 1 \leq x \leq 3 \\ -2 & \text{if } x > 3 \end{cases}$$

**Differentiate.**

7. 
$$y = \frac{x-1}{2x+1}$$

8. 
$$y = -5x \tan 3x$$

**Differentiate using implicit differentiation.**

9. 
$$2xy^2 - 3y + x^3 = -5$$

10. 
$$\cos x - \cos y = 2x^2y^2$$

**Differentiate using logarithmic differentiation.**

11. 
$$y = \frac{7x\sqrt{x+4}}{3e^x}$$

12. 
$$y = \frac{\sqrt[4]{x^5}(2x^2+3x)}{(x^3-4)^2}$$

**Find the equation of the tangent line at the given value or point.**

13. 
$$y = 2 \cot x \text{ at } x = \frac{\pi}{4}$$

14. 
$$x^2y - y^2 = -6 \text{ at } (-1, -2)$$

**Find the intervals on which  $f(x)$  is increasing, decreasing, concave up, and concave down.**

15. 
$$f(x) = x^3 - 2x^2 + x - 1$$

16. 
$$f(x) = 2xe^x$$

**Find the values of  $c$  that satisfies the Mean Value Theorem.**

17. 
$$f(x) = 3x^2 - 7x + 1 \text{ on } [0, 2]$$

18. 
$$f(x) = \sqrt{2x+3} \text{ on } [-1, 11]$$

**Find the absolute minimum and maximum on the given interval.**

19.  $f(x) = x^4 - 8x^2 + 1$  on  $[-2, 1]$

20.  $f(x) = -\sin 3x$ ,  $\left[-\frac{\pi}{2}, 0\right]$

**(Calculator is necessary). Solve each related rate problem. Round answers to the nearest thousandths.**

21. A six foot tall man is walking towards a light pole at a speed of 3 ft per second. The light 12 ft up casts a shadow behind the man. How fast is the tip of the shadow moving toward the light pole when he is 7 ft from the light pole?

22. A cone with a diameter of 14 ft and a height of 28 ft is being filled with oil at a rate of 2 ft<sup>3</sup> per second. How fast is the level of the oil rising in the cone when the height reaches 10 ft?

**(Calculator is necessary). Solve each optimization problem.**

23. Determine the dimensions of a box of maximum volume that can be made from a piece of material 8.5" x 11". The box is to be made by cutting square pieces from the corners and folding up the sides. The box will not have a top.

24. Given a length of string  $L = 74$  inches, construct a circle and a square such that the sum of the areas is a maximum.

**Integrate.**

25.  $\int_0^4 \frac{x}{x^2 + 4} dx$

26.  $\int \frac{2 \ln \sqrt{y}}{y} dy$

27.  $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sin x}{\cos x - 1} dx$

28.  $\int_2^5 |x - 4| dx$

**Find the following estimations using a Riemann Sum.**

29.  $y = x^2 + 2$  on  $[-1, 3]$  when  $n = 2$  using right hand endpoints.

30.  $y = -x^2 - 3$  on  $[-1, 2]$  when  $n = 3$  using left hand endpoints.

**Find the area between the curve and the x-axis of the following function.**

31.  $y = -x^2 - x + 2$  from  $[-1, 2]$ .

**Answers:**

1)  $-9$

2)  $\frac{1}{10}$

3)  $\frac{7}{5}$

4)  $\infty$

5) Continuous at all values of  $x$  except  $-1/2$  and  $-4$ 6) Continuous at all values of  $x$  except  $3$ 

7)  $\frac{3}{(2x+1)^2}$

8)  $-5 \tan(3x) - 15x \sec^2(3x)$

9)  $\frac{dy}{dx} = \frac{2y^2 + 3x^2}{3 - 4xy}$

10)  $\frac{dy}{dx} = \frac{4xy^2 + \sin x}{\sin y - 4x^2y}$

11)  $\frac{dy}{dx} = \left( \frac{1}{x} + \frac{1}{2(x+4)} - 1 \right) \frac{7x\sqrt{x+4}}{3e^x}$

12)  $\frac{dy}{dx} = \left( \frac{5}{4x} + \frac{4x+3}{2x^2+3x} - \frac{6x^2}{x^3-4} \right) \frac{\sqrt[4]{x^5}(2x^2+3x)}{(x^3-4)^2}$

13)  $y = -4 \left( x - \frac{\pi}{4} \right) + 2$

14)  $y = -\frac{4}{5}(x+1) - 2$

15)  $Inc: \left( -\infty, \frac{1}{3} \right) \cup (1, \infty)$   $Dec: \left( \frac{1}{3}, 1 \right)$   $CU: \left( \frac{2}{3}, \infty \right)$   $CD: \left( -\infty, \frac{2}{3} \right)$

16)  $Inc: (-1, \infty)$   $Dec: (-\infty, -1)$   $CU: (-2, \infty)$   $CD: (-\infty, -2)$

17)  $c = 1$

18)  $c = 3$

19)  $Max: f(0) = 1$   $Min: f(-2) = -15$

20)  $Max: f(0) = 0$  &  $f\left(-\frac{\pi}{6}\right) = 0$   $Min: f\left(-\frac{\pi}{2}\right) = -1$

21)  $6 \text{ ft / sec}$

22)  $\frac{8}{25\pi} \text{ ft / sec}$

23)  $h = 1.585"$ ,  $w = 5.33"$ ,  $l = 7.83"$

24) radius of the circle is  $5.18$  in.length of the side of the square is  $10.36$  in.

25)  $\frac{1}{2} \ln 5$

26)  $2(\ln \sqrt{y})^2 + C$

27) 0

28)  $\frac{5}{2}$

29) 28

30) -11

31)  $\frac{31}{6}$