HW14
HPC A

Name key

Cumulative Review A

Find and simplify the difference quotient for the given functions.

1) $f(x) = -4x^2 + 2x - 1$

$$\frac{-4(x+h)^2 + 2(x+h) - 1 - (-4x^2+2x-1)}{h}$$

$$\frac{-4(x^2+2xh+h^2) + 2x + 2h - 1 + 4x^2 - 2x + 1}{h}$$

$$\frac{-8xh - 4h^2 + 2h}{h}$$

$$-8x - 4h + 2$$

2) $f(x) = 3x^2 - 2x - 5$

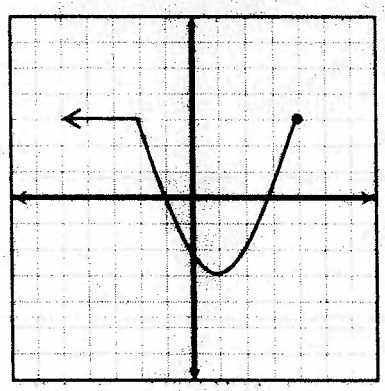
$$\frac{3(x+h)^2 - 2(x+h) - 5 - (3x^2 - 2x - 5)}{h}$$

$$\frac{3(x^2+2xh+h^2) - 2x - 2h - 5 - 3x^2 + 2x + 5}{h}$$

$$\frac{6xh + 3h^2 - 2h}{h}$$

$$6x + 3h - 2$$

Use the graph of f to the right to answer the following problems. Use interval notation where appropriate. (Assume that all tick marks on the graph are 1 unit apart.)



3. Domain $(-\infty, 4]$
4. Range $[-3, 3]$
5. x-intercept(s) $1, 2.8$
6. y-intercept(s) -2
7. Interval(s) at which f is increasing $(1, 4)$
8. Interval(s) at which f is decreasing $(-2, 1)$
9. Interval(s) at which f is constant $(-\infty, -2)$
10. What is the relative minimum of f? -3
11. At what number does f have a relative minimum? 1
12. $f(-6) =$ 3
13. At what values of x is $f(x) < 0$? $(-1, 2.8)$
14. Is f even, odd, or neither? neither

Given functions f and g, perform the indicated operations and find the domain of the resulting function.

15) $f(x) = \frac{7}{x+1}$, $g(x) = \frac{x}{2x+1}$

Find $(f \circ g)(x)$.

$$f(g(x)) = \frac{7}{\frac{x}{2x+1} + 1}$$

$$\frac{7(2x+1)}{x + 2x + 1} =$$

$$\frac{14x+7}{3x+1}$$

$x \neq -\frac{1}{3}, -\frac{1}{2}$

$$(-\infty, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, -\tfrac{1}{3}) \cup (-\tfrac{1}{3}, \infty)$$

16) $f(x) = x^2 + 2x$, $g(x) = x^2 - 2x - 8$

Find $\left(\frac{f}{g}\right)(x)$.

$$\frac{x^2+2x}{x^2-2x-8}$$

$$\frac{x(x+2)}{(x-4)(x+2)}$$

$$\frac{x}{x-4}$$

$x \neq 4, -2$

$$(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$$

Given functions f and g, perform the indicated operation..

17) $f(x) = 3x^2 - x + 5, \quad g(x) = -x - 6$

Find $(fg)(x)$.

$(3x^2 - x + 5)(-x - 6)$

$-3x^3 - 18x^2 + x^2 + 6x - 5x - 30$

$\boxed{-3x^3 - 17x^2 + x - 30}$

18) $f(x) = \sqrt{4x + 3}$

Find $f^{-1}(x)$.

$x = \sqrt{4y + 3}$

$x^2 = 4y + 3$

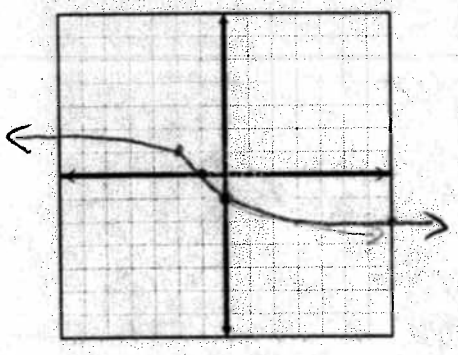
$x^2 - 3 = 4y$

$y = \frac{x^2 - 3}{4}$

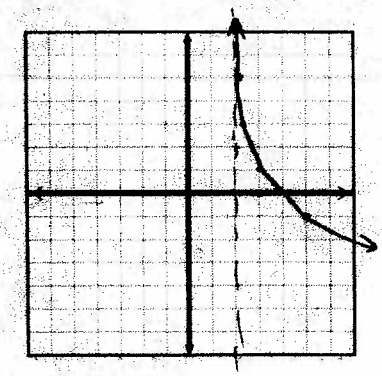
$\boxed{f^{-1}(x) = \frac{1}{4}x^2 - \frac{3}{4} \quad \text{or} \quad f^{-1}(x) = \frac{x^2 - 3}{4}}$

Graph the following functions.

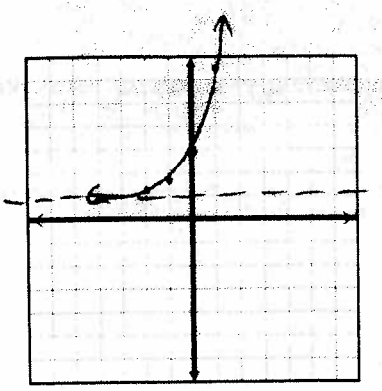
19) $f(x) = -\sqrt[3]{x + 1}$



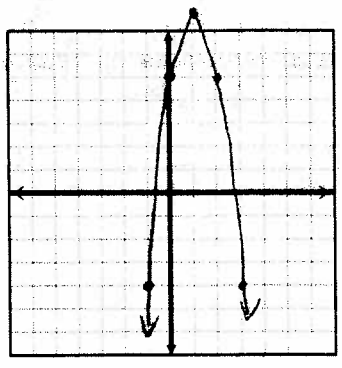
20) $f(x) = -2\log_3(x - 2) + 1$



21) $f(x) = 2e^x + 1$



22) $f(x) = -3x^2 + 6x + 5$



Graph the following functions. State the a) symmetry, b) y-intercept, c) x-intercept(s), d) vertical asymptote(s), e) horizontal or slant asymptote, f) hole (if any)

23) $f(x)=\dfrac{x}{2x^2-1}$

x-int = 0

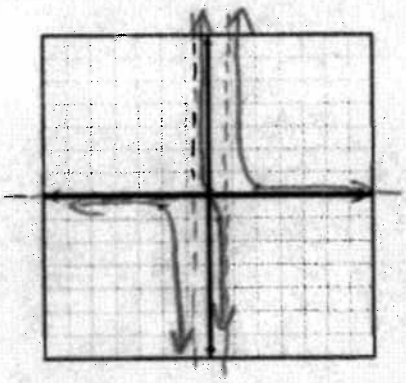
y-int. = 0

VA: $x=\pm\sqrt{\frac{1}{2}}$

$x\approx\pm 0.7$

HA: $y=0$

x	y
−2	−2/7
0.5	−1
2	2/7



24) $f(x)=\dfrac{x^2-3x-4}{x+3}$ $\quad \dfrac{(x-4)(x+1)}{x+3}$

x-int: 4, −1

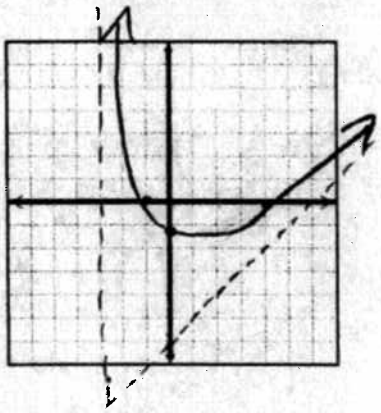
y-int: $-4/3$

VA: $x=-3$

SA:

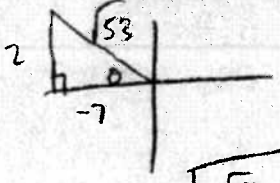
$$x+3\overline{)\,x^2-3x-4}\qquad x-6,\ x^2+3x,\ -6x-18$$

x	y
−4	−24



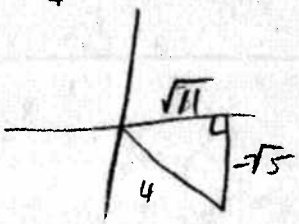
Find the exact value of the indicated trigonometry function of θ.

25) If $\cot\theta=-\dfrac{7}{2}$; $\sin\theta>0$. Find $\sec\theta$



$\sec\theta=-\dfrac{\sqrt{53}}{7}$

26) If $\sin\theta=-\dfrac{\sqrt{5}}{4}$; $\tan\theta<0$. Find $\cot\theta$



$\dfrac{\sqrt{11}}{-\sqrt{5}}=-\dfrac{\sqrt{55}}{5}$

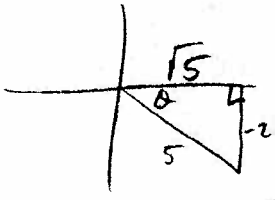
Find the exact value of the expression if possible. Do not use a calculator.

27) $\cot^{-1}\left[\sin\left(-\dfrac{\pi}{2}\right)\right]$

$\cot^{-1}(-1)$

$-\dfrac{\pi}{4}$

28) $\cot\left[\sin^{-1}\left(-\dfrac{2\sqrt{5}}{5}\right)\right]$



$\dfrac{\sqrt{5}}{-2\sqrt{5}}=-\dfrac{1}{2}$

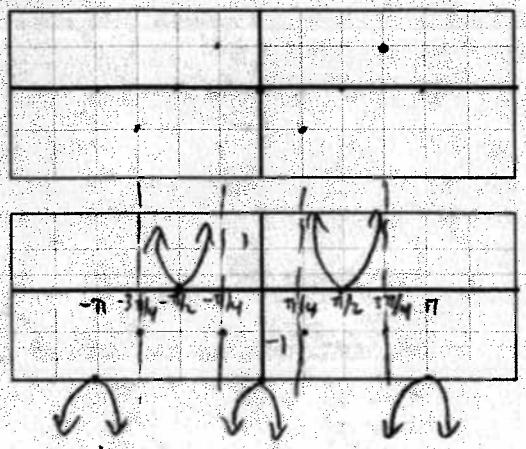
Graph one or more periods. Be sure to label all units.

29) $y = -\csc\left(2x + \frac{\pi}{2}\right) - 1$

amplitude = 1

period = π

shift = Left $\pi/4$ down 1

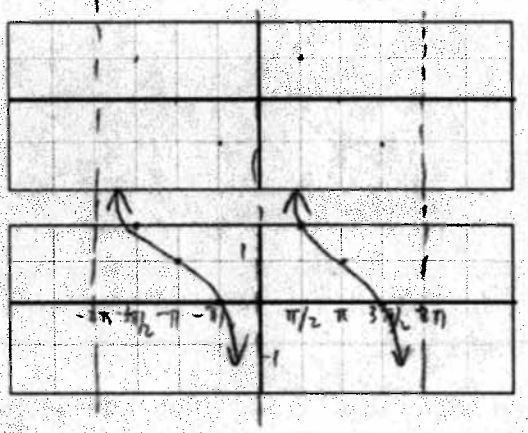


30) $y = \cot\left(\frac{x}{2} - \pi\right) + 1$

amplitude = 1

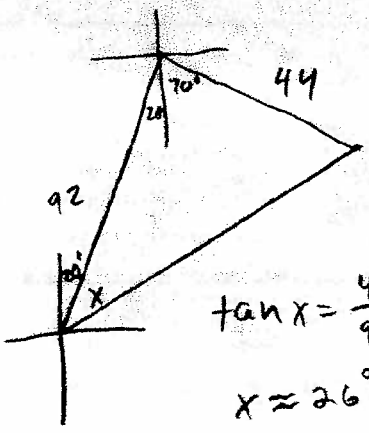
period = 2π

shift = right 2π up 1



Solve. Round answers to the nearest degree.

31) A plane leaves an airport at a heading of N20°E for 92 miles. It then changes course and heads S70°E for 44 miles and lands at a new airport. What is the bearing from the original airport to the new airport?

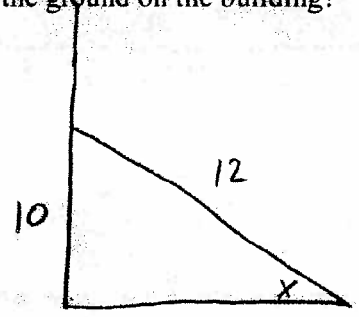


$\tan x = \frac{44}{92}$

$x \approx 26°$

N46°E

32) A 12 foot ladder is attached to a 100 foot building somewhere between the top of the building and the ground. What angle will the ladder need to form with the ground if it needs to rest 10 feet above the ground on the building?



$\sin x = \frac{10}{12}$

$x \approx 56°$

Verify each identity.

33) $\dfrac{1+\tan\theta}{\sin\theta+\cos\theta}=\sec\theta$

$$\frac{\cos\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)}{\cos\theta(\sin\theta+\cos\theta)}$$

$$\frac{\cos\theta+\sin\theta}{\cos\theta(\sin\theta+\cos\theta)}$$

$$\frac{1}{\cos\theta}\ \checkmark$$

34) $\dfrac{\sin\theta}{1-\cot\theta}+\dfrac{\cos\theta}{1-\tan\theta}=\sin\theta+\cos\theta$

$$\frac{\sin\theta(\sin\theta)}{\sin\theta\left(1-\frac{\cos\theta}{\sin\theta}\right)}+\frac{\cos\theta(\cos\theta)}{\cos\theta\left(1-\frac{\sin\theta}{\cos\theta}\right)}=$$

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$$

$$\sin\theta+\cos\theta\ \checkmark$$

Find the exact value. You may use any of your formulas.

35) $\tan\dfrac{7\pi}{12}$

$$\tan\left(\frac{7\pi}{6}\div 2\right)$$

$$\frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$\frac{1+\frac{\sqrt{3}}{2}}{-1/2}=-2-\sqrt{3}$$

36) $\cos 112.5^\circ$

$$\cos\frac{225}{2}=$$

$$-\sqrt{\frac{1+\cos 225}{2}}$$

$$-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$-\frac{\sqrt{2-\sqrt{2}}}{2}$$

Find all solutions in the interval $[0,2\pi)$.

37) $\sqrt{2}\csc 3x+1=3$

$$\csc 3x=\frac{2}{\sqrt{2}}$$

$$\sin 3x=\frac{\sqrt{2}}{2}$$

$$\sin u=\frac{\sqrt{2}}{2}$$

$$u=\frac{\pi}{4},\frac{3\pi}{4},\frac{9\pi}{4},\frac{11\pi}{4},\frac{17\pi}{4},\frac{19\pi}{4}$$

$$u=3x$$

$$x=\frac{\pi}{12},\frac{3\pi}{4},\frac{\pi}{4},\frac{11\pi}{12},\frac{17\pi}{12},\frac{19\pi}{12}$$

38) $4\cos x=\sin 2x$

$$4\cos x=2\sin x\cos x$$

$$4\cos x-2\sin x\cos x=0$$

$$2\cos x(2-\sin x)=0$$

$\cos x=0$ $\qquad$ ~~$\sin x=2$~~

$$x=\frac{\pi}{2},\frac{3\pi}{2}$$