Spring Semester Honors Pre-Calculus Final Exam Review

1. A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes. If P is the point (15, 230) on the graph of V, estimate the slope of the tangent line at P by averaging the slopes of two secant lines, passing through P and the points on the graph with t = 10 and t = 20.

t (min)	5	10	15	20	25	30
V (gal)	625	450	230	174	28	0

2. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after *t* minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute. The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with *t* = 38 and *t* = 42.

t (min)	36	38	40	42	44
Heartbeats	2500	2660	2880	2980	3010

3. The displacement (in feet) of a certain particle moving in a straight line is given by $s(t)=\frac{t^3}{6}$ where t is measured in seconds. Find the instantaneous velocity when t = 5.

4. The slope of the tangent line to the graph of the exponential function $y = 2^x$ at the point (0, 1) is $\lim_{x\to 0}\frac{2^x-1}{x}$ Estimate the slope to three decimal places.

5. Evaluate the limit $\lim_{x\to 4}\left(5x^2+4x+8\right)$

6. Evaluate the limit $\lim_{x\to -4}\left|x+4\right|$

7. Find the point at which the given function is discontinuous. $f(x)=\begin{cases}\frac{1}{x-4}, & x\neq 4\\ 6, & x=4\end{cases}$

8. For what value of the constant c is the function *f* continuous on $(-\infty,\infty)$? $f(x)=\begin{cases}cx+30, & x\leq 4\\ cx^2-6, & x>4\end{cases}$

9. If an arrow is shot upward on the moon, with a velocity of 67 m/s its height (in meters) after *t* seconds is given by $H(t)=67t-0.77t^2$. With what velocity will the arrow hit the moon?

10. The position function of a particle is given by $s(t)=t^3-6t^2-2t,\ \ t\geq 0.$ When does the particle reach a velocity of 94 m/s?

11. If a tank holds 5000 gallons of water, and that water can drain from the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$V(t)=5000\left(1-\frac{t}{40}\right)^2$. Find the rate at which water is draining from the tank after 6 minutes.

12. If $f(x)=\frac{x}{\ln x}$, find $f(e^2)$.

13. A man starts walking north at 7 ft/s from a point P. Five minutes later a woman starts walking south at 2 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 10 min after the woman starts walking? Round the result to the nearest hundredth.

14. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 27 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base? Round the result to the nearest hundredth if necessary.

15. At noon, ship A is 130 km west of ship B. Ship A is sailing south at 39 km/h and ship B is sailing north at 26 km/h. How fast is the distance between the ships changing at 5:00 p.m.? Round the result to the nearest thousandth if necessary.

16. Evaluate the limit: $\lim_{x\to 3}\left(\frac{x^3-1}{x^2-4}\right)$

17. Evaluate the limit: $\lim_{x\to\infty}\left(\frac{5x^2-3x+4}{3x^2+5x-6}\right)$

18. Find the limit: $\lim_{x\to\infty}\left(\frac{1}{2x+9}\right)$

19. Let P and Q be polynomials. Find $\lim_{x\to\infty}\frac{P(x)}{Q(x)}$ if the degree of P is greater than the degree of Q.

20. Find an equation of the tangent line to the parabola $y=x^2+5x$ at the point (9, 126).

21. A curve has equation y = f(x). Write an expression for the slope of the secant line through the points P(1, f(1)) and Q(x, f(x)).

22. Differentiate the function $f(t)=\frac{1}{3}t^6-8t^4+t$

23. Differentiate the function $s(r)=4\pi r^2$

24. Differentiate the function $y = \frac{x^2 + 2x + 7}{\sqrt{x}}$

25. Differentiate $f(x) = x^5 e^x$

26. Differentiate $y = \frac{e^{3x}}{1 + 3x}$

27. Differentiate $R(t) = \left(3t + e^{2t}\right)\left(1 - \sqrt{t}\right)$

28. Find the equation of the tangent line to the curve $y = 6\tan x$ at the point (π/4, 6).

29. Differentiate $g(x) = x^2 \cos x$

30. Differentiate $g(x) = 9\sec x + \tan x$

31. Differentiate $y = \frac{\tan x - 5}{\sec x}$

32. Find the equation of the tangent line to the curve $y = \sec x - 2\cos x$ at the point (π/3, 1).

33. Find y' by implicit differentiation $xy + 8x + 6x^2 = 7$

34. Find the equation of the tangent line to the curve $2x^2 + 4y^2 = 6$ at the point (1, 1).

35. Find the equation of the tangent line to the curve $y^2 = x^3(10 - x)$ at the point (1, 3).

36. If $f(x) = 9\cos x + \sin^2 x$ find f '(x) and f "(x).

37. Find f ''' (x) if $y = \sqrt{2x + 8}$

38. Differentiate $f(x) = \cos(\ln 3x)$

39. Differentiate $f(t) = \frac{4 + \ln t}{5 - \ln t}$

40. Differentiate $G(u) = \ln\sqrt{\frac{2u + 4}{2u - 4}}$

41. Use logarithmic differentiation to find the derivative of the function $y = \sqrt[5]{\frac{x^2 + 1}{x^2 - 1}}$

42. Use logarithmic differentiation to find the derivative of the function $y = x^{4x}$

43. Find the critical numbers of the function $y = 9x^2 + 36x$

44. Find the absolute minimum values of $y = 3x^2 - 30x + 10$

45. Find the absolute minimum value of $y = 3x^2 + \frac{6}{x}$ on the interval from [0, 6].

46. Find the critical numbers of the function $F(x) = x^{4/5}(x-3)^2$

47. Verify that the function satisfies the hypotheses of The Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of The Mean Value Theorem.
$f(x) = 7x^2 + 6x + 2, \quad [-9, 9]$

48. Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem. $f(x) = \sin 9\pi x, \quad \left[-\frac{2}{9}, \frac{2}{9}\right]$

49. How many points of inflection are on the graph of the function $f(x) = -17x^3 + 11x^2 - 20x - 13$?

50. Find the inflection points of the following function $f(x) = -4x + 3 - 2\sin x, \quad 0 < x < 3\pi$

51. Find the intervals on which the following function *f* is increasing $f(x) = x^3 - 108x + 8$

52. Estimate to the hundredth the area from 1 to 5 under the graph of $f(x) = \frac{4}{x}$ using four approximating rectangles and right endpoints.

53. Estimate to the hundredth, the area from 0 to 5 under the graph of $f(x) = 64 - x^2$ using five approximating rectangles with right endpoints.

54. Evaluate the Riemann sum for $f(t) = 5 - t^2, \ 0 \le t \le 2,$ with four subintervals, taking the sample points to be right endpoints.

55. Evaluate the integral by interpreting it in terms of areas. $\int_1^3 (3 + 4x)\, dx$

56. If $\int_2^{10} f(x)dx = 4$ and $\int_9^{10} f(x)dx = 0.9,$ find $\int_2^9 f(x)dx.$

57. Evaluate the integral $\int_{2}^{6} \frac{x^2+5}{\sqrt{x}}\,dx$

58. Evaluate the integral $\int_{0}^{\pi/6} \frac{5+\cos^2\theta}{\cos^2\theta}\,d\theta$

59. Find the indefinite integral $\int x\left(2+7x^4\right)dx$

60. Find the indefinite integral $\int (6-t)\left(4+t^2\right)dt$

61. Evaluate the integral $\int_{-\pi/8}^{\pi/8} \frac{x^2\sin x}{4+x^6}\,dx$

62. Evaluate the integral $\int_{e^{16}}^{e^{81}} \frac{dx}{x\sqrt{\ln x}}$

63. Find the indefinite integral $\int 8x\left(x^2+3\right)dx$

64. Find the indefinite integral $\int \frac{2+6x}{\sqrt{8+2x+3x^2}}dx$

65. Find the indefinite integral $\int \sec^8 x \tan x dx$

66. Find the area between the curves $y=7x+6$ and $y=8x^2$

67. Find the area between the curves $y=7x^2$ and $y=x^2+1$

68. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y=\frac{1}{x}$ from $x = 3$ to $x = 10$

69. Find the volume of the solid obtained by rotating the region bounded by $y=\sqrt[3]{x}$ and $y=x$ about the line $y = 1$