Notes 14 **Name** _____

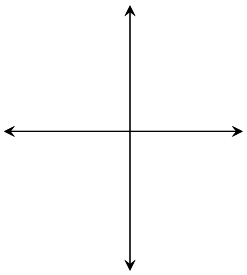
Double-Angle and Half-Angle Formulas (5.3)

Double-Angle Formulas

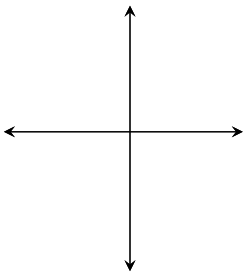
1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad = 2\cos^2\theta - 1 \qquad = 1 - 2\sin^2\theta$
3. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Using the Double-Angle Formulas (Forward): Use the given information to find the exact value.

1) Find $\sin 2\theta$ if $\sin\theta = \dfrac{12}{13}$, θ lies in quadrant II.



2) Find $\tan 2\theta$ if $\cot\theta = 3$, θ lies in quadrant III.



Using the Double-Anlge Formulas (Backwards): Find the exact value of each expression.

3) $1 - 2\sin^2\dfrac{\pi}{12}$

4) $\dfrac{2\tan 22.5^\circ}{1-\tan^2 22.5^\circ}$

Verifying Identities Using the Sum or Difference Formulas: Verify the following identities.

5) $\dfrac{\cos 2x}{\cos^2 x} = 1 - \tan^2 x$

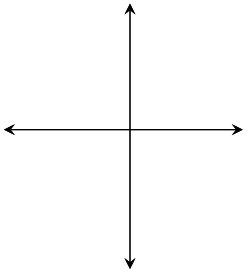
6) $\sin 2\theta = \dfrac{2\cot\theta}{1+\cot^2\theta}$

Half-Angle Formulas

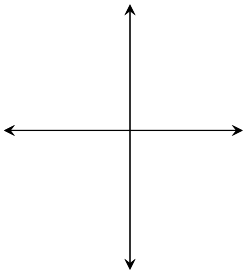
1. $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$
2. $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$
3. $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \qquad = \frac{1-\cos x}{\sin x} \qquad = \frac{\sin x}{1+\cos x}$

Using the Half-Angle Formulas (Forward): Use the given information to find the exact value.

7) Find $\cos\frac{\theta}{2}$ if $\tan\theta = -\frac{8}{15}$, θ lies in quadrant II.



8) Find $\tan\frac{\theta}{2}$ if $\sec\theta = -3$, θ lies in quadrant III.



Find the Value: Use a half-angle formula to find the exact value of each expression.

9) $\sin 22.5^\circ$

10) $\tan\frac{3\pi}{8}$