Notes 22 **Name** _____

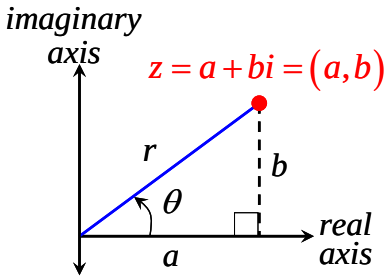
Complex Numbers in Polar Form; DeMoivre's Theorem (6.5)

Polar Form of a Complex Number

The complex number $z = a + bi$ is written in polar form as

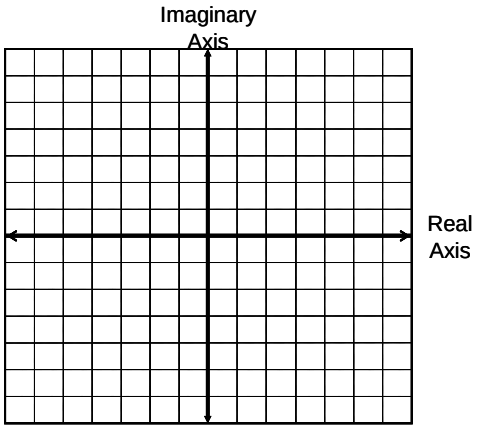
$$z = r(\cos\theta + i\sin\theta)$$

$$a = r\cos\theta$$
$$b = r\sin\theta$$
$$r = \sqrt{a^2 + b^2}$$
$$\tan\theta = \frac{b}{a}$$



r is called the modulus and θ is called the argument

Writing a Complex Number in Polar Form: Plot each complex number. Then write the complex number in polar form. You may express the argument in degrees or radians.



1. $z = 2 - 2i$

2. $z = -2 + 2\sqrt{3}i$

Writing a Complex Number in Rectangular Form: Write each complex number in rectangular form. If necessary, round to the nearest tenth.

3. $z = 10(\cos 210^\circ + i\sin 210^\circ)$

4. $z = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

Product of Two Complex Numbers in Polar Form

Given $z_1 = r_1\cos\theta_1 + i\sin\theta_1$ and $z_2 = r_2\cos\theta_2 + i\sin\theta_2$

$$z_1 z_2 = r_1 r_2\left[\cos\left(\theta_1+\theta_2\right)+i\sin\left(\theta_1+\theta_2\right)\right]$$

Quotient of Two Complex Numbers in Polar Form

Given $z_1 = r_1\cos\theta_1 + i\sin\theta_1$ and $z_2 = r_2\cos\theta_2 + i\sin\theta_2$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos\left(\theta_1-\theta_2\right)+i\sin\left(\theta_1-\theta_2\right)\right]$$

Finding Products and Quotients of Complex Numbers in Polar Form: Find the product and quotient of the complex numbers. Leave answers in polar form.

5. $z_1 = 4\left(\cos 15^\circ + i\sin 15^\circ\right)$
   $z_2 = 7\left(\cos 25^\circ + i\sin 25^\circ\right)$

6. $z_1 = 5\left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right)$
   $z_2 = 10\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)$

DeMoivre's Theorem

$$z^n = \left[r\left(\cos\theta + i\sin\theta\right)\right]^n$$
$$= r^n\left(\cos n\theta + i\sin n\theta\right)$$

DeMoivre's Theorem for Finding Complex Roots

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta+2\pi k}{n}\right)+i\sin\left(\frac{\theta+2\pi k}{n}\right)\right] \text{ or}$$

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta+360^\circ k}{n}\right)+i\sin\left(\frac{\theta+360^\circ k}{n}\right)\right]$$

where $k = 0,\ 1,\ 2,\ldots,\ n-1$

Finding the Power of a Complex Number: Find the indicated power of the complex number. Write the answer in rectangular form.

7. $\left[2\left(\cos 40^\circ + i\sin 40^\circ\right)\right]^3$

Finding the Roots of a Complex Number: Find ***all*** the complex roots. Write the roots in rectangular form.

8. The complex third roots of $8\left(\cos\pi + i\sin\pi\right)$