

10.1

# Sequences & Summation Notation

*Objective: How to use factorial, sequence & summation notation apply it to sequences & series.*



# Sequences:

**Finite Sequence:** domain (input values) is the set of consecutive integers, and has a final term; if domain is not specified, it starts with 1

ex: 3,6,9,12,15

**Infinite Sequence:** sequence that does not end

ex: 3,6,9,12,15,...

$n$  = number of the term

$a_n$  = value of the  $n$ th term (general term)

# Writing Terms of a Sequence from the General Term

Find the first four terms of the sequence given by:

1.  $a_n = 3 + 4(-1)^n$

$$a_1 = 3 + 4(-1)^1 = -1$$

$$a_2 = 3 + 4(-1)^2 = 7$$

$$a_3 = 3 + 4(-1)^3 = -1$$

$$a_4 = 3 + 4(-1)^4 = 7$$

$-1, 7, -1, 7$

2.  $a_n = \frac{(-1)^n}{2n-1}$

$$a_1 = \frac{(-1)^1}{2(1)-1} = -1$$

$$a_2 = \frac{(-1)^2}{2(2)-1} = \frac{1}{3}$$

$$a_3 = \frac{(-1)^3}{2(3)-1} = -\frac{1}{5}$$

$$a_4 = \frac{(-1)^4}{2(4)-1} = \frac{1}{7}$$

$-1, \frac{1}{3}, -\frac{1}{5}, \frac{1}{7}$

## Using a Recursion Formula

Find the first four terms of the sequence given by:

3.  $a_1 = -4$  and  $a_n = 2a_{n-1} - 3$  for  $n \geq 2$

First  
Term

General  
Term

Previous  
Term

$$a_2 = 2(-4) - 3 = -11$$

$$a_3 = 2(-11) - 3 = -25$$

$$a_4 = 2(-25) - 3 = -53$$

$$a_5 = 2(-53) - 3 = -109$$

**-11, -25, -53, -109**

## Using a Recursion Formula

Find the first four terms of the sequence given by:

4.  $a_1 = 5$  and  $a_n = 3a_{n-1} - 1$  for  $n \geq 2$

$$a_2 = 14$$

$$a_3 = 41$$

$$a_4 = 122$$

$$a_5 = 365$$

14, 41, 122, 365

# Series (Sums):

-when the terms of a sequence are added

**Finite Series:** has an end; specific number of terms being added

$$\text{ex: } 3+6+9+12+15$$

**Infinite Series:** series that does not end

$$\text{ex: } 3+6+9+12+15+\dots$$

## Summation Notation (Sigma Notation):

Upper Limit of Summation

Lower Limit of Summation

Notation

$$\sum_{n=1}^5 3n = 3 + 6 + 9 + 12 + 15$$

Find the summation:

$$5. \sum_{k=1}^4 (2k^2 - 1)$$

$$a_1 = 2(1)^2 - 1 = 1$$

$$a_2 = 2(2)^2 - 1 = 7$$

$$a_3 = 2(3)^2 - 1 = 17$$

$$a_4 = 2(4)^2 - 1 = 31$$

$$1 + 7 + 17 + 31 =$$

$$\boxed{56}$$

$$6. \sum_{i=0}^4 \frac{(-1)^{i+1}}{(i+1)!}$$

$$a_0 = \frac{(-1)^{0+1}}{(0+1)!} = -1 \quad a_1 = \frac{(-1)^{1+1}}{(1+1)!} = \frac{1}{2}$$

$$a_2 = \frac{(-1)^{2+1}}{(2+1)!} = -\frac{1}{6} \quad a_3 = \frac{(-1)^{3+1}}{(3+1)!} = \frac{1}{24}$$

$$a_4 = \frac{(-1)^{4+1}}{(4+1)!} = -\frac{1}{120}$$

$$-1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} =$$

$$-\frac{120}{120} + \frac{60}{120} - \frac{20}{120} + \frac{5}{120} - \frac{1}{120} =$$

$$-\frac{76}{120} = \boxed{-\frac{19}{30}}$$