

1.9 and 9.3
Circles and Parabolas

The Distance Formula

◆ The distance d between the points (x_1, y_1) and (x_2, y_2) is as follows:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Midpoint Formula

- ◆ The midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is as follows:

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- ◆ Each coordinate of M is the mean of the corresponding coordinates of A and B.

Find the Distance and Midpoint

1. $(8, -4)$ and $(5, -2)$

$$d = \sqrt{13} \quad MP = \left(\frac{13}{2}, -3 \right)$$

2. $(3, -1)$ and $(-5, -2)$

$$d = \sqrt{65} \quad MP = \left(-1, -\frac{3}{2} \right)$$

Standard Form of a Circle

- ◆ The standard form of the equation of a circle with center at (h, k) and radius r is as follows:

$$(x - h)^2 + (y - k)^2 = r^2$$

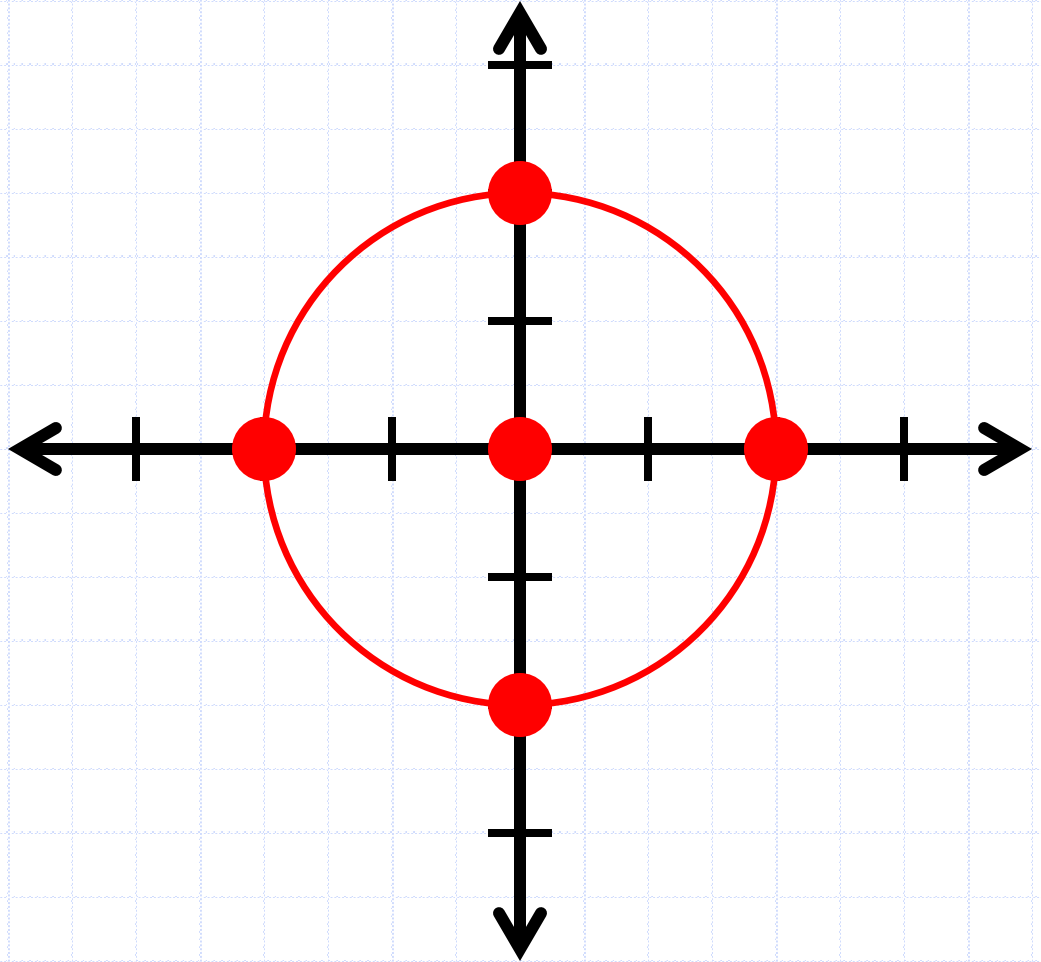
#3 Graph

$$y^2 = 4 - x^2$$

$$x^2 + y^2 = 4$$

$$C = (0, 0)$$

$$r = 2$$

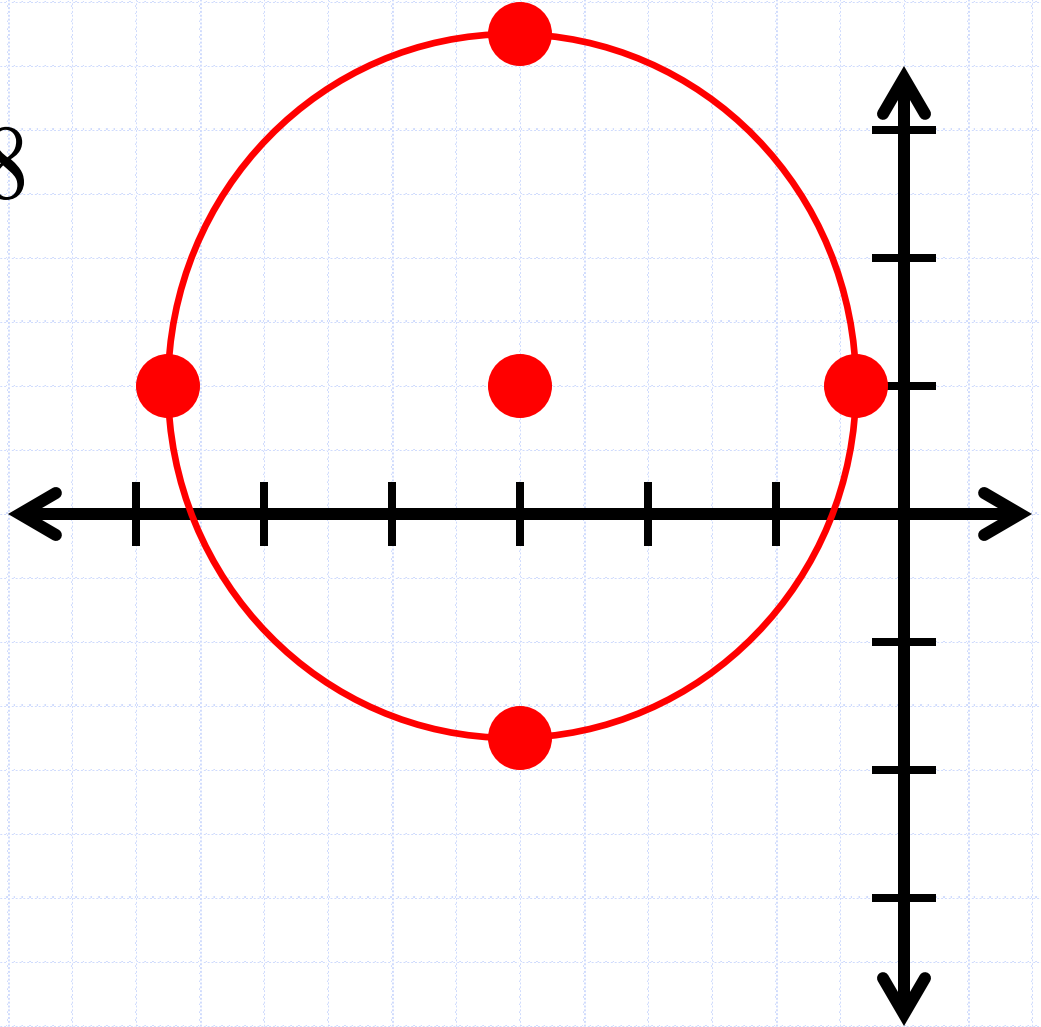


#4 Graph

$$(x+3)^2 + (y-1)^2 = 8$$

$$C = (-3, 1)$$

$$r = 2.8$$



#5

◆ $(2, -1)$ is on a circle centered at the point $(-1, -2)$. Write the equation of the circle.

Find the distance:

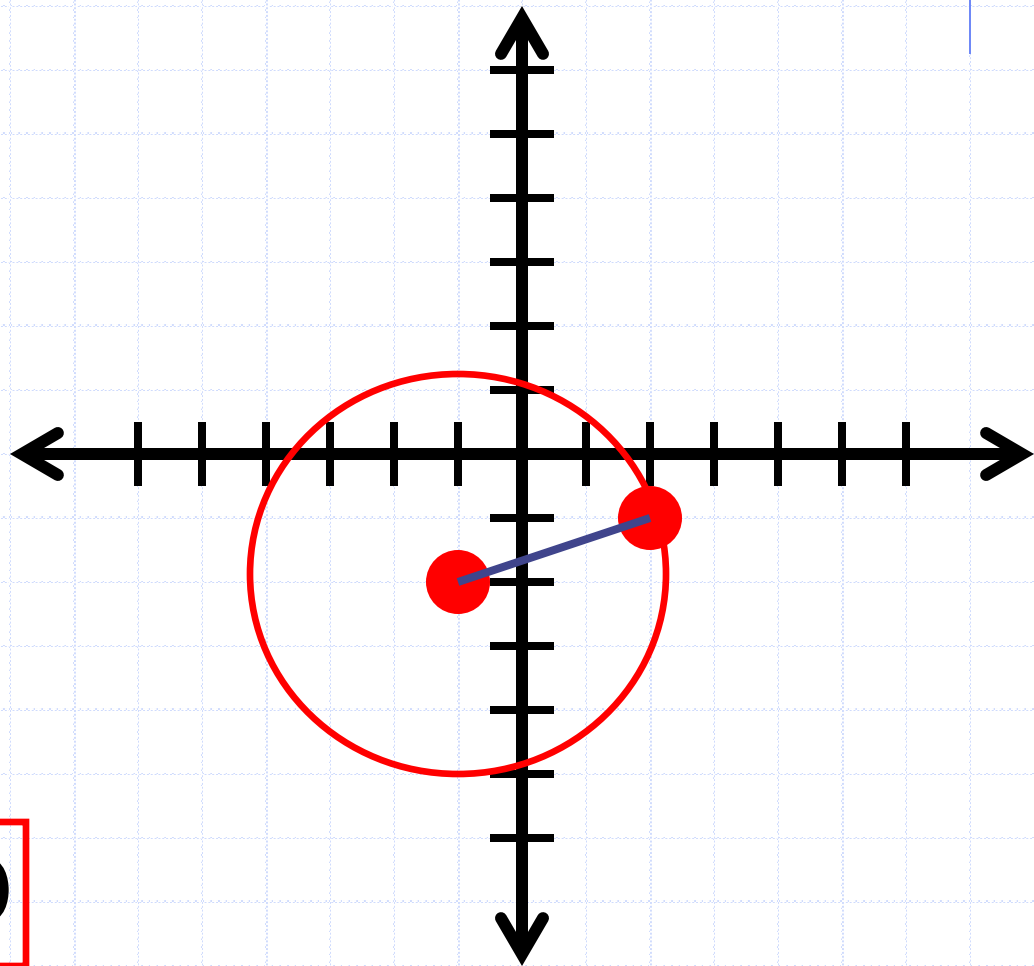
$(2, -1)$ and $(-1, -2)$

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$r = \sqrt{(2 + 1)^2 + (-1 + 2)^2}$$

$$r = \sqrt{10}$$

$$(x + 1)^2 + (y + 2)^2 = 10$$



#6

◆ $(5, 1)$ is on a circle centered at the point $(1, 3)$. Write the equation of the circle.

Find the distance:

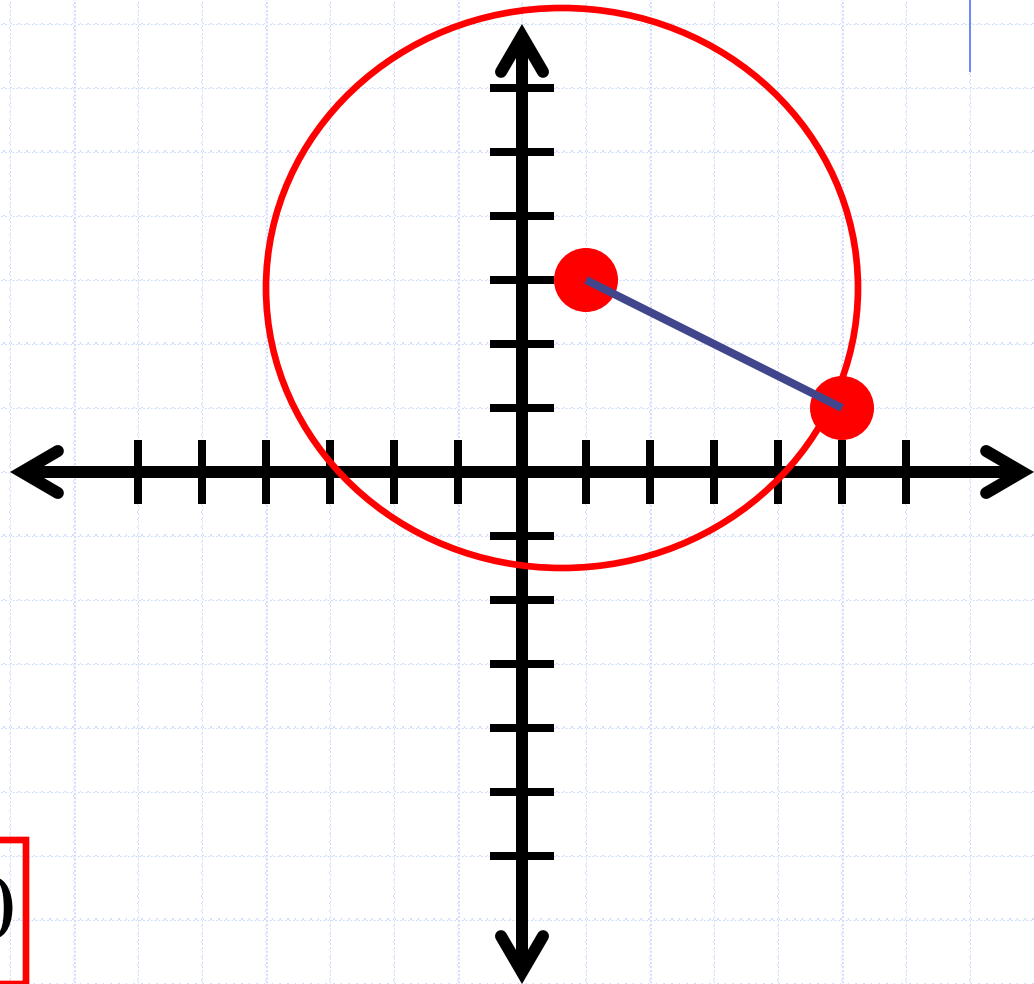
$(5, 1)$ and $(1, 3)$

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$r = \sqrt{(5 - 1)^2 + (1 - 3)^2}$$

$$r = \sqrt{20}$$

$$(x - 1)^2 + (y - 3)^2 = 20$$



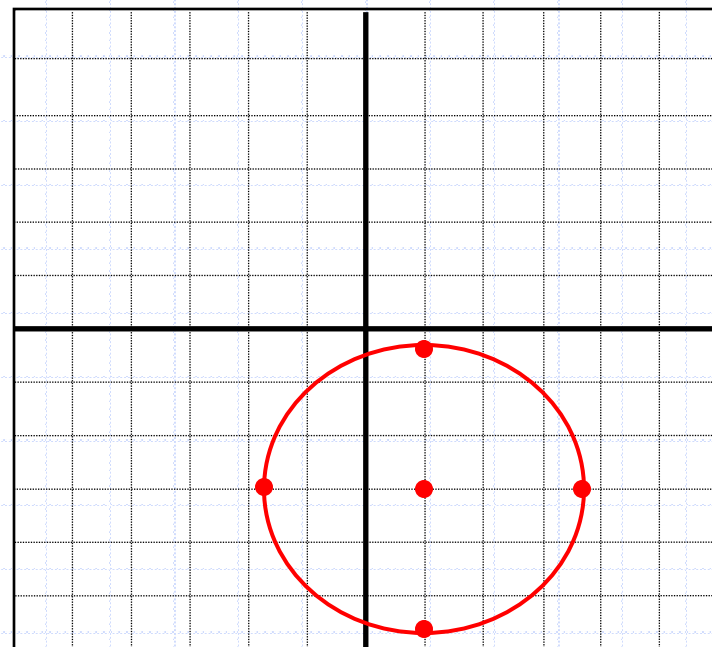
Writing a Circle in Standard Form

$$7. x^2 + y^2 - 2x + 6y + 3 = 0$$

$$x^2 - 2x + y^2 + 6y = -3$$

$$x^2 - 2x + \underline{1} + y^2 + 6y + \underline{9} = -3 + \underline{1} + \underline{9}$$

$$(x-1)^2 + (y+3)^2 = 7$$



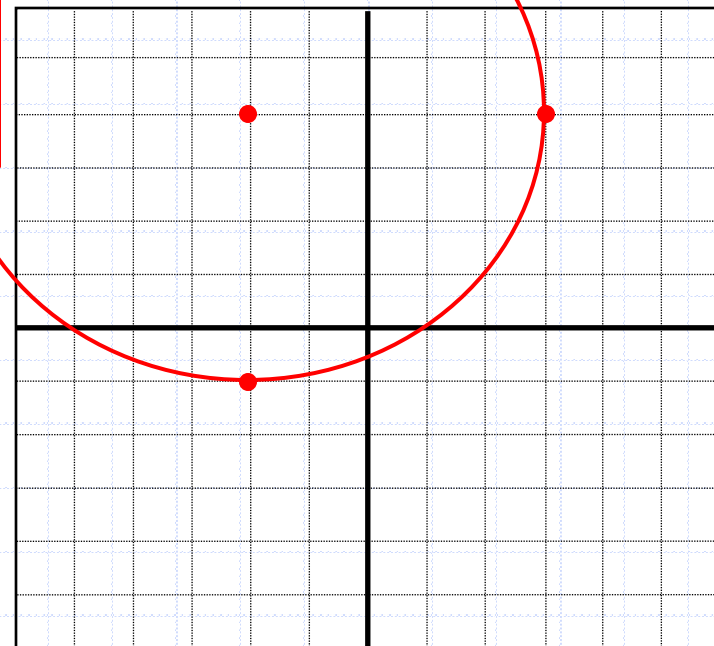
Writing a Circle in Standard Form

8. $x^2 + y^2 + 4x - 8y - 5 = 0$

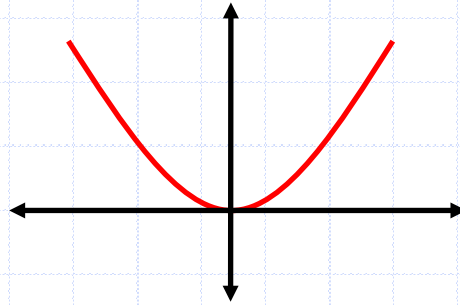
$$x^2 + 4x + y^2 - 8y = 5$$

$$x^2 + 4x + \underline{4} + y^2 - 8y + \underline{16} = 5 + \underline{4} + \underline{16}$$

$$(x + 2)^2 + (y - 4)^2 = 25$$



Vertical Axis of Symmetry



$$(x - h)^2 = 4p(y - k)$$

Vertex: (h, k)

P:

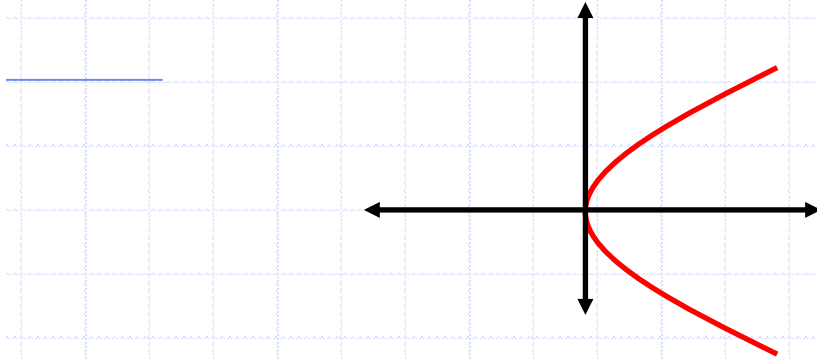
+ : opens up

- : opens down

Focus: p units away from the vertex, inside the parabola.

Directrix: the *horizontal* line $-p$ units away from the vertex

Horizontal Axis of Symmetry



$$(y - k)^2 = 4p(x - h)$$

Vertex: (h, k)

P:

+ : opens right

- : opens left

Focus: p units away from the vertex, inside the parabola.

Directrix: the *vertical* line $-p$ units away from the vertex

#1 Graph the parabola. Identify the vertex, focus, and directrix.

$$-2x^2 = 16y$$

Write in standard form:

$$-2x^2 = 16y$$

$$x^2 = -8y \quad \text{Vertical}$$

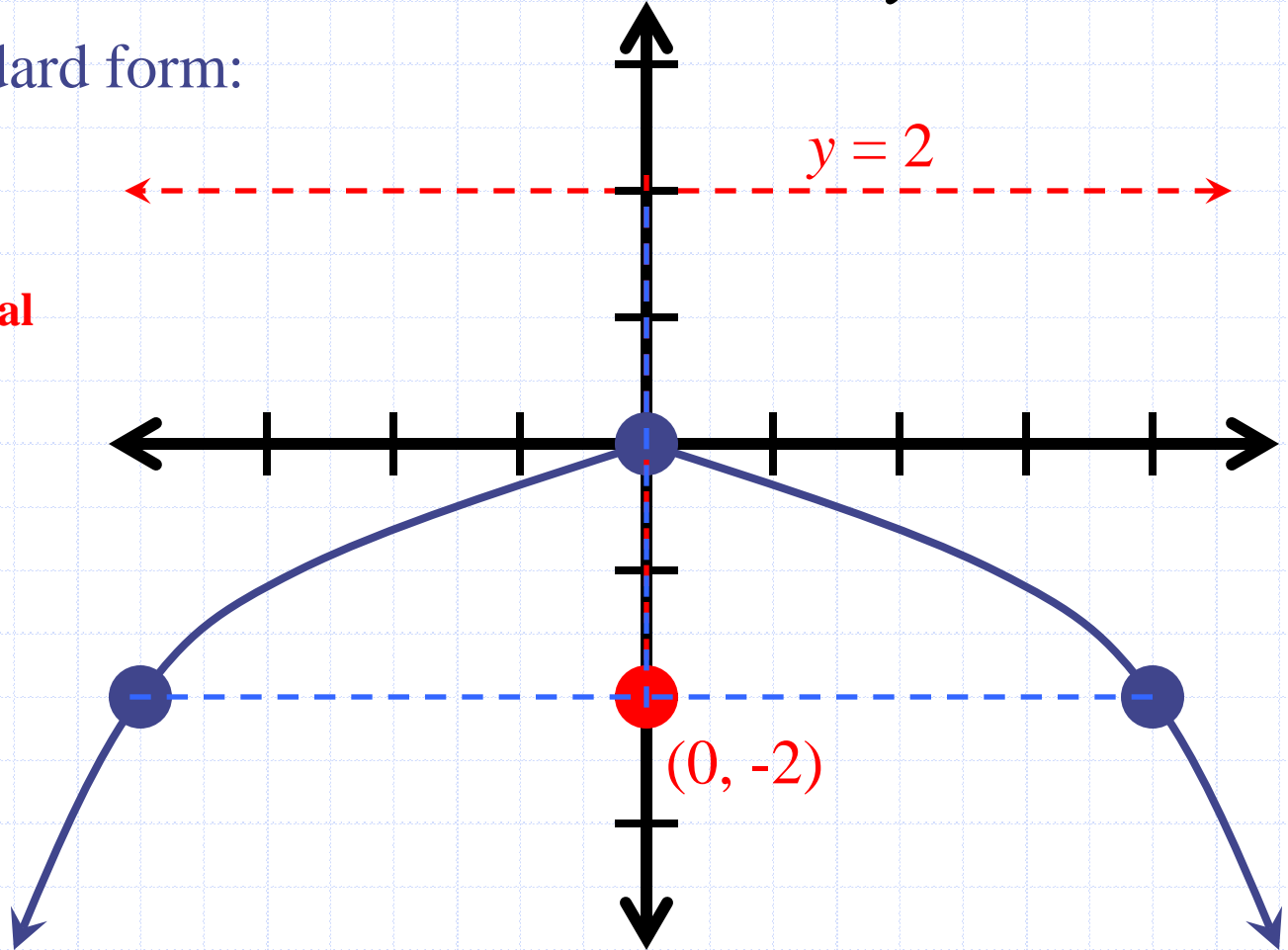
$$\text{Vertex } (0, 0)$$

$$4p = -8$$

$$p = -2$$

$$\text{Focus } (0, -2)$$

$$\text{Directrix } y = 2$$



#2 Graph the parabola. Identify the vertex, focus, and directrix.

Write in standard form:

$$6x = y^2$$

$$y^2 = 6x \quad \text{Horizontal}$$

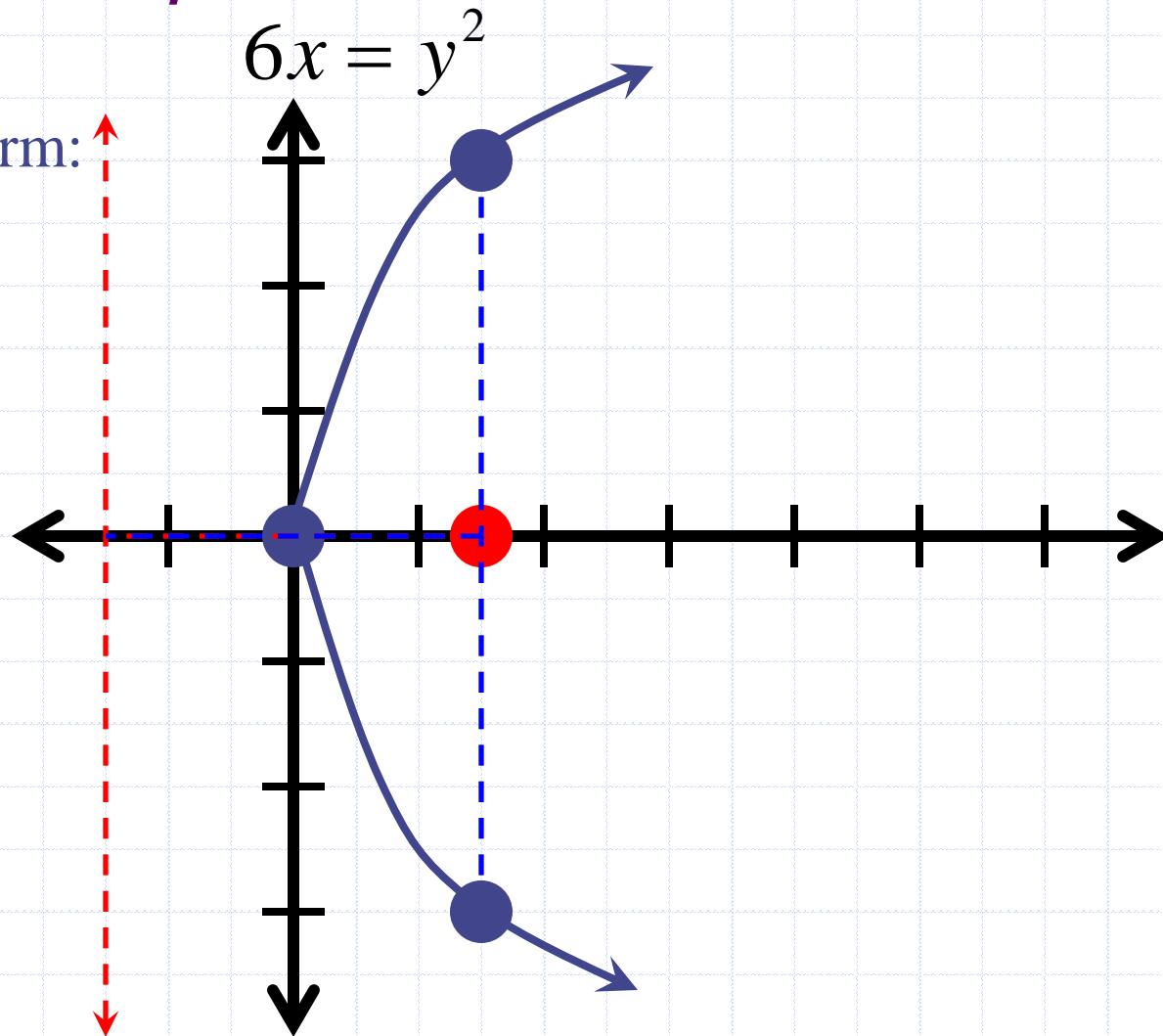
$$\text{Vertex } (0, 0)$$

$$4p = 6$$

$$p = \frac{3}{2}$$

$$\text{Focus } \left(\frac{3}{2}, 0 \right)$$

$$\text{Directrix: } x = -\frac{3}{2}$$



#3 Graph the parabola. Identify the vertex, focus, and directrix.

$$(y - 2)^2 = 4(x - 3) \text{ Horizontal}$$

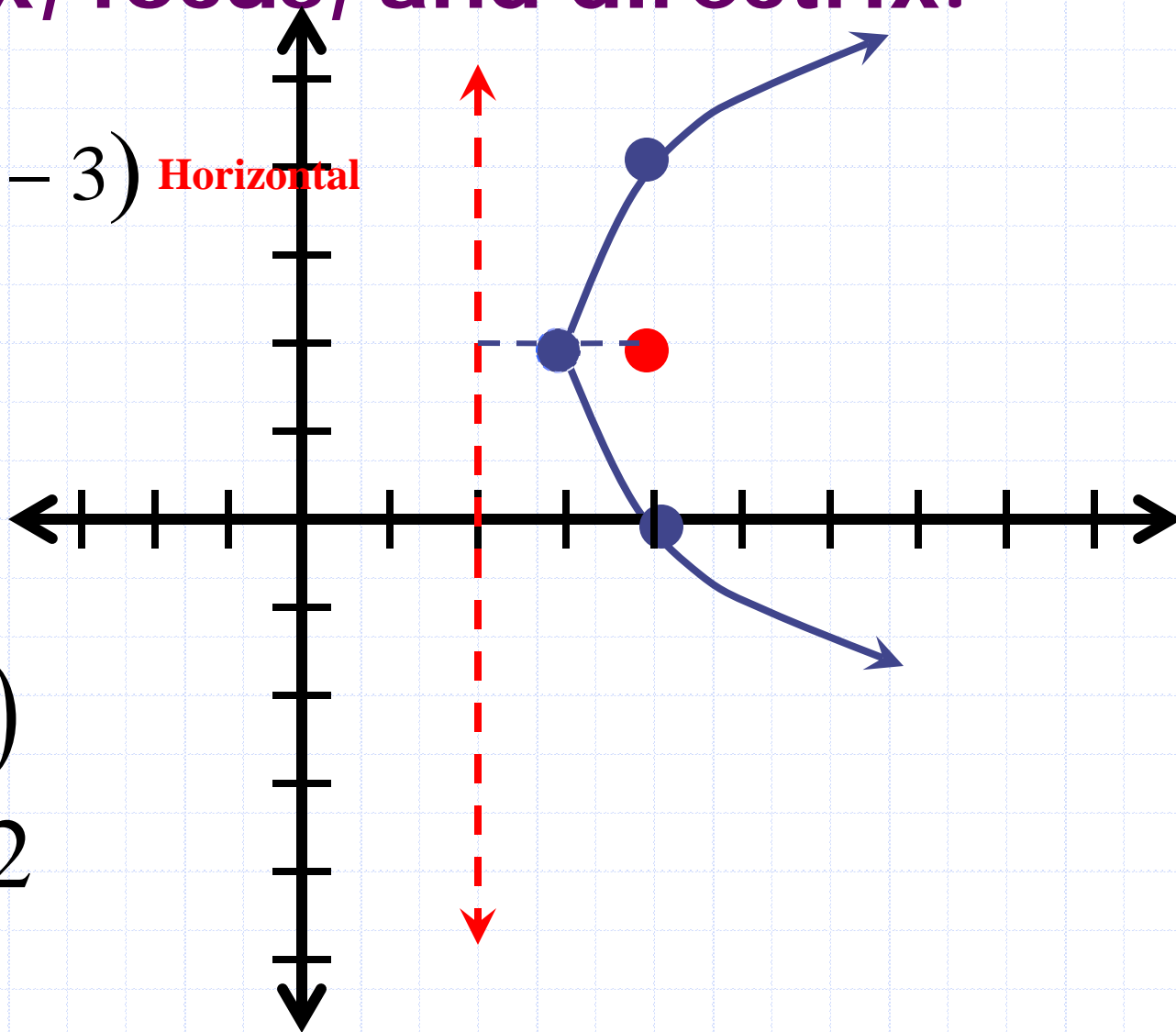
Vertex $(3, 2)$

$$4p = 4$$

$$p = 1$$

Focus $(4, 2)$

Directrix $x = 2$



#4 Graph the parabola. Identify the vertex, focus, and directrix.

$$(x + 1)^2 = -6(y - 2) \quad \text{Vertical}$$

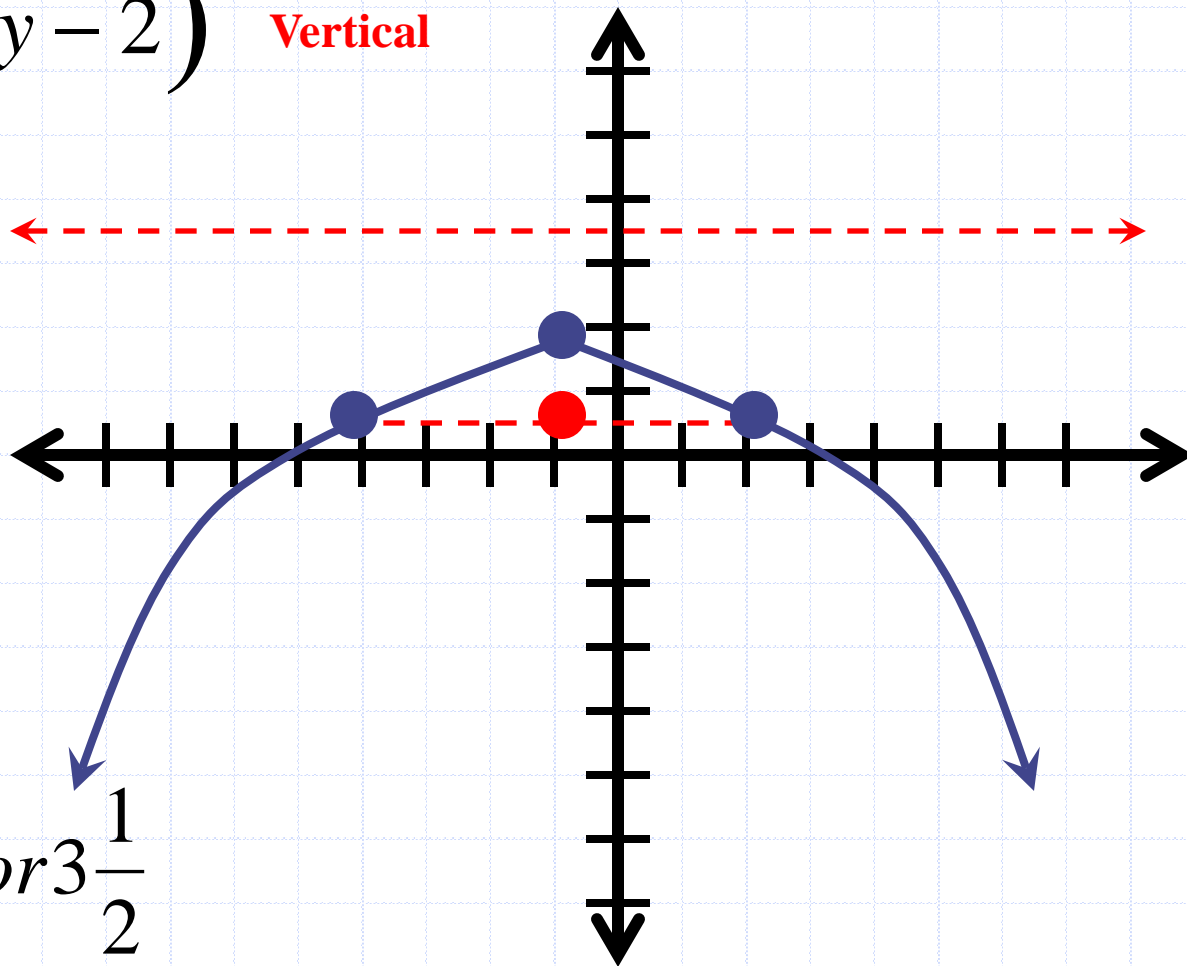
$$\text{Vertex } (-1, 2)$$

$$4p = -6$$

$$p = -\frac{3}{2}$$

$$\text{Focus } \left(-1, \frac{1}{2}\right)$$

$$\text{Directrix } y = \frac{7}{2} \text{ or } 3\frac{1}{2}$$



Writing an equation of a parabola in standard form

$$5. \quad y^2 - 2y + 16x - 31 = 0$$

$$y^2 - 2y = -16x + 31$$

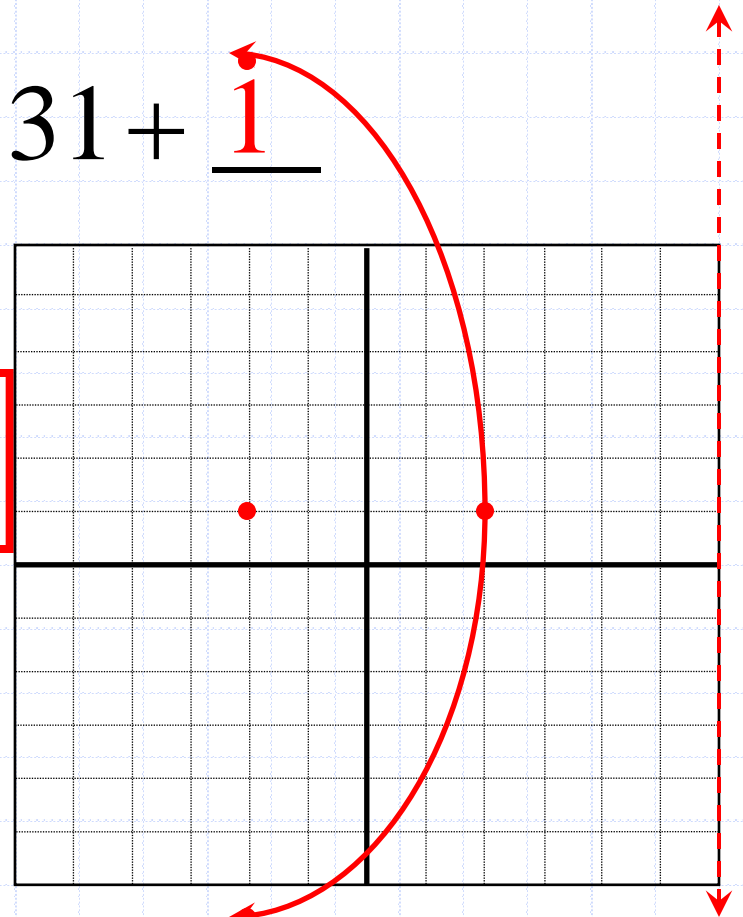
$$y^2 - 2y + \underline{1} = -16x + 31 + \underline{1}$$

$$(y - 1)^2 = -16x + 32$$

$$(y - 1)^2 = -16(x - 2)$$

Horizontal

Vertex: (2, 1)



Writing an equation of a parabola in standard form

6. $x^2 + 10x - 4y + 1 = 0$

$$x^2 + 10x = 4y - 1$$

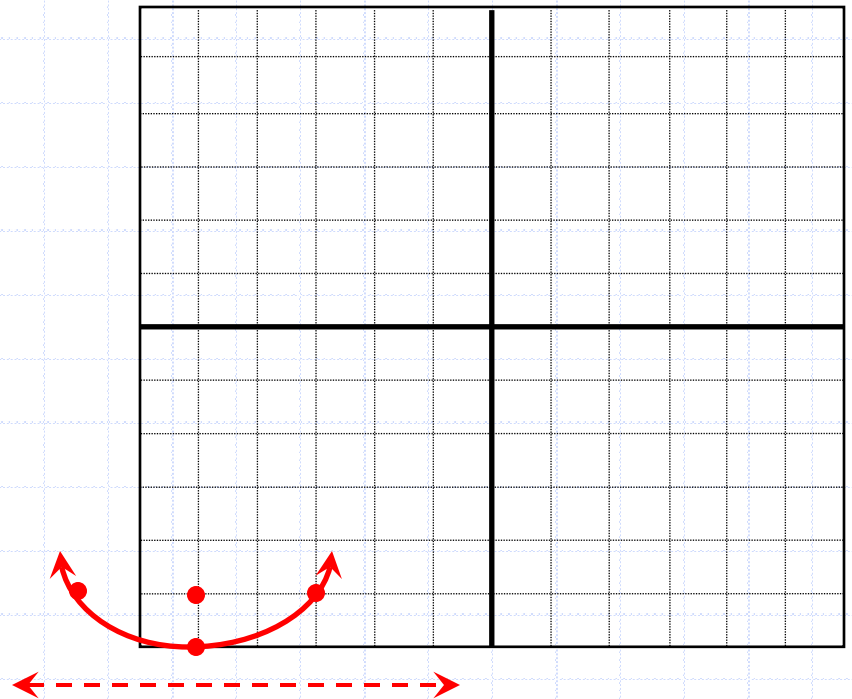
$$x^2 + 10x + \underline{25} = 4y - 1 + \underline{25}$$

$$(x + 5)^2 = 4y + 24$$

$$(x + 5)^2 = 4(y + 6)$$

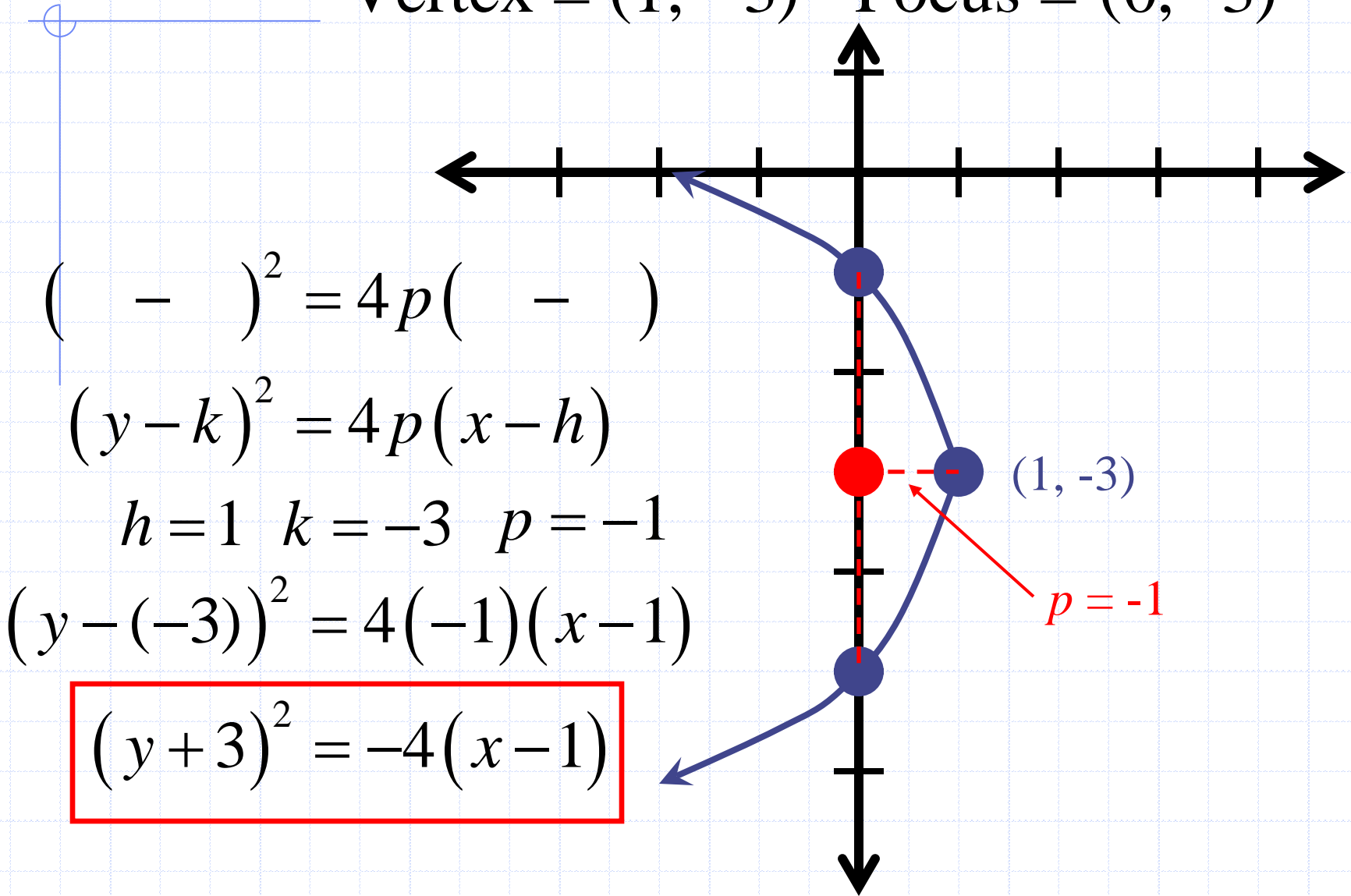
Vertical

Vertex: (-5, -6)



#7 Write an equation of the parabola

Vertex = $(1, -3)$ Focus = $(0, -3)$



$$\left(\quad - \quad \right)^2 = 4p \left(\quad - \quad \right)$$

$$(y - k)^2 = 4p(x - h)$$

$$h = 1 \quad k = -3 \quad p = -1$$

$$(y - (-3))^2 = 4(-1)(x - 1)$$

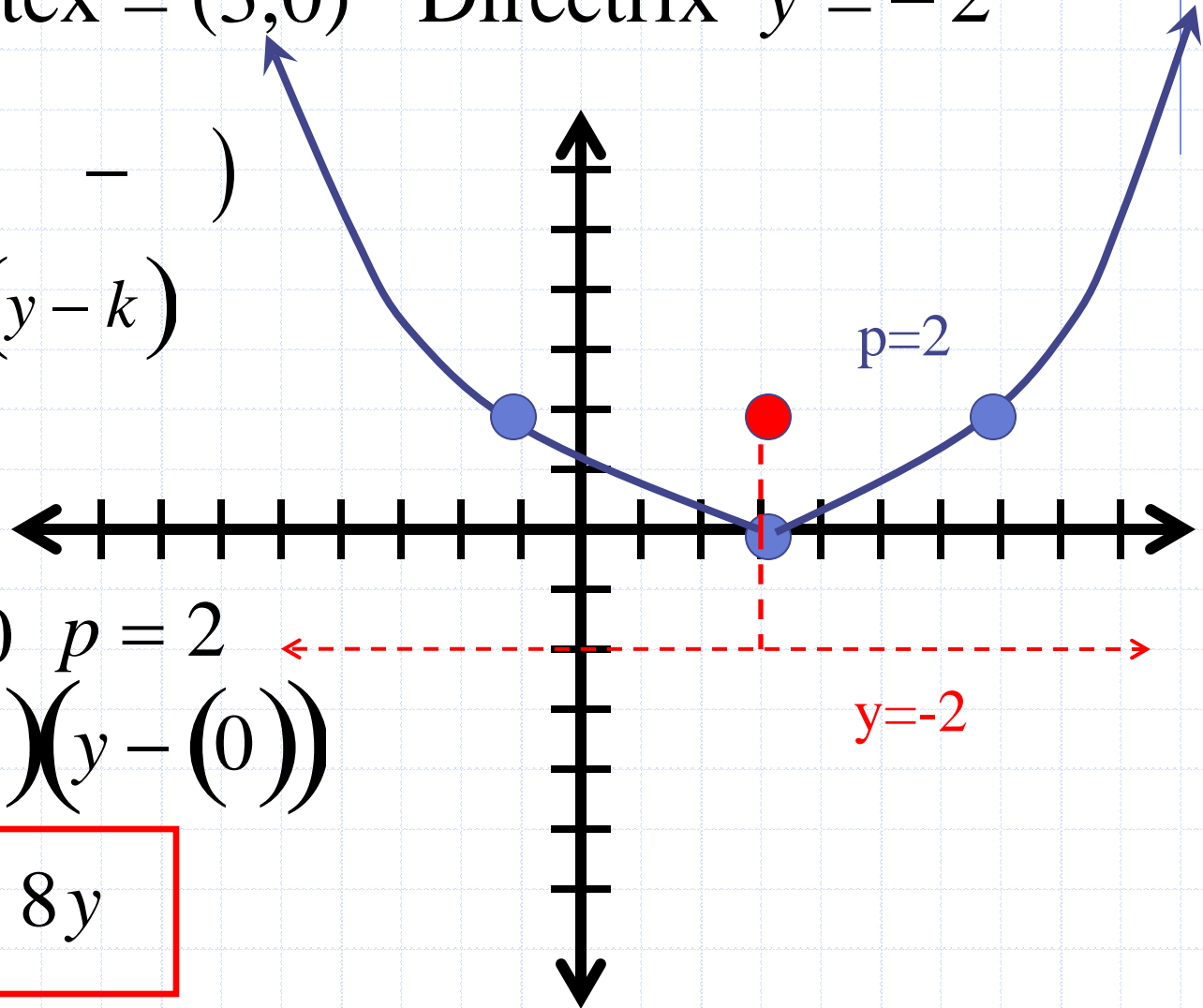
$$(y + 3)^2 = -4(x - 1)$$

#8 Write an equation of the parabola

Vertex = (3,0) Directrix $y = -2$

$$\left(\quad - \quad \right)^2 = 4p \left(\quad - \quad \right)$$

$$(x - h)^2 = 4p(y - k)$$



$$h = 3 \quad k = 0 \quad p = 2$$

$$(x - (3))^2 = 4(2)(y - (0))$$

$$(x - 3)^2 = 8y$$