9.1
Ellipses

Standard Forms of an Ellipse

- The standard form of the equation of an ellipse with center at (*h*, *k*).

Horizontal

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Vertices $(h\pm a,k)$

Co-Vertices $(h,k\pm b)$

Vertical

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Vertices $(h,k\pm a)$

Co-Vertices $(h\pm b,k)$

$$Foci:\ c^2=a^2-b^2$$

Foci $(h\pm c,k)$

Foci $(h,k\pm c)$

#1 Graph the ellipse

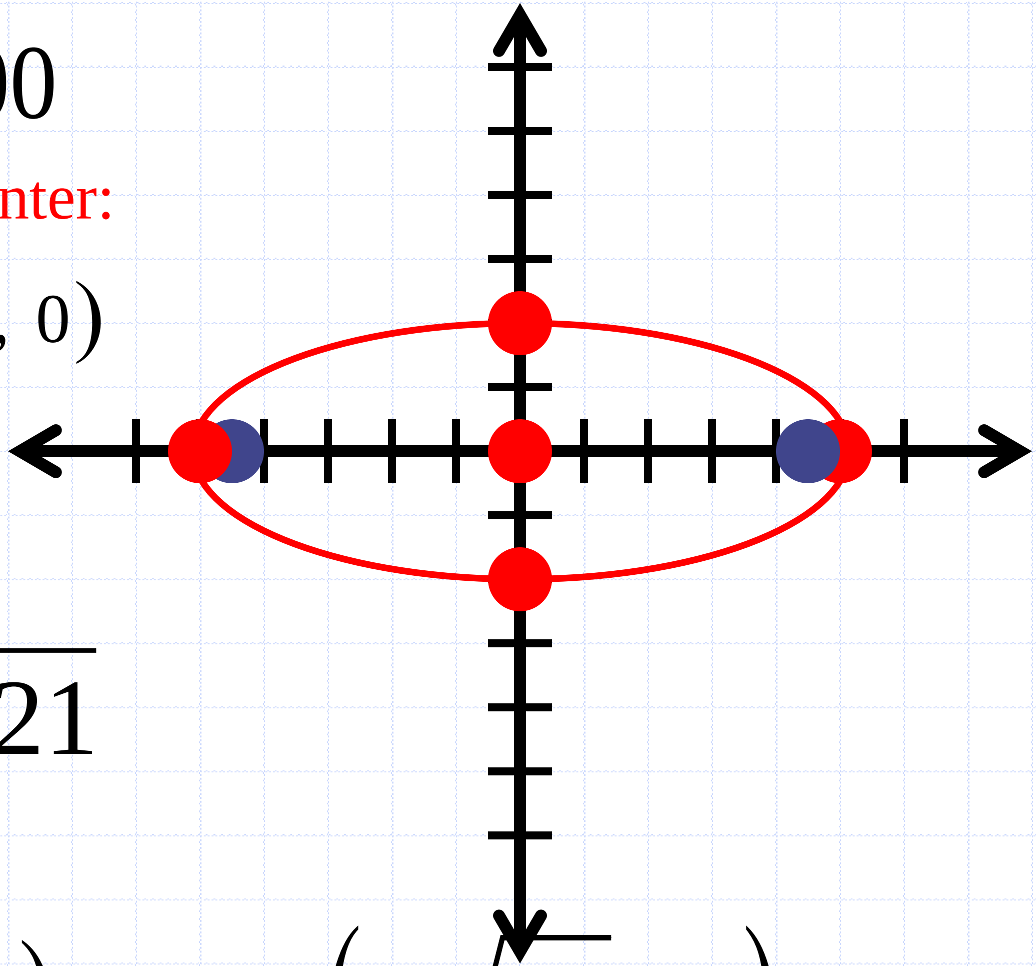
Label the Center Vertices, Co-Verticies, and Foci.

$$4x^2 + 25y^2 = 100$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

Center:

$(0,\ 0)$



$$a = 5 \quad b = 2$$

$$c = \sqrt{25 - 4} = \sqrt{21}$$

Vertices $\left(\pm 5, 0\right)$

Co-Vertices $\left(0, \pm 2\right)$

Focus $\left(\pm\sqrt{21},\ 0\right)$

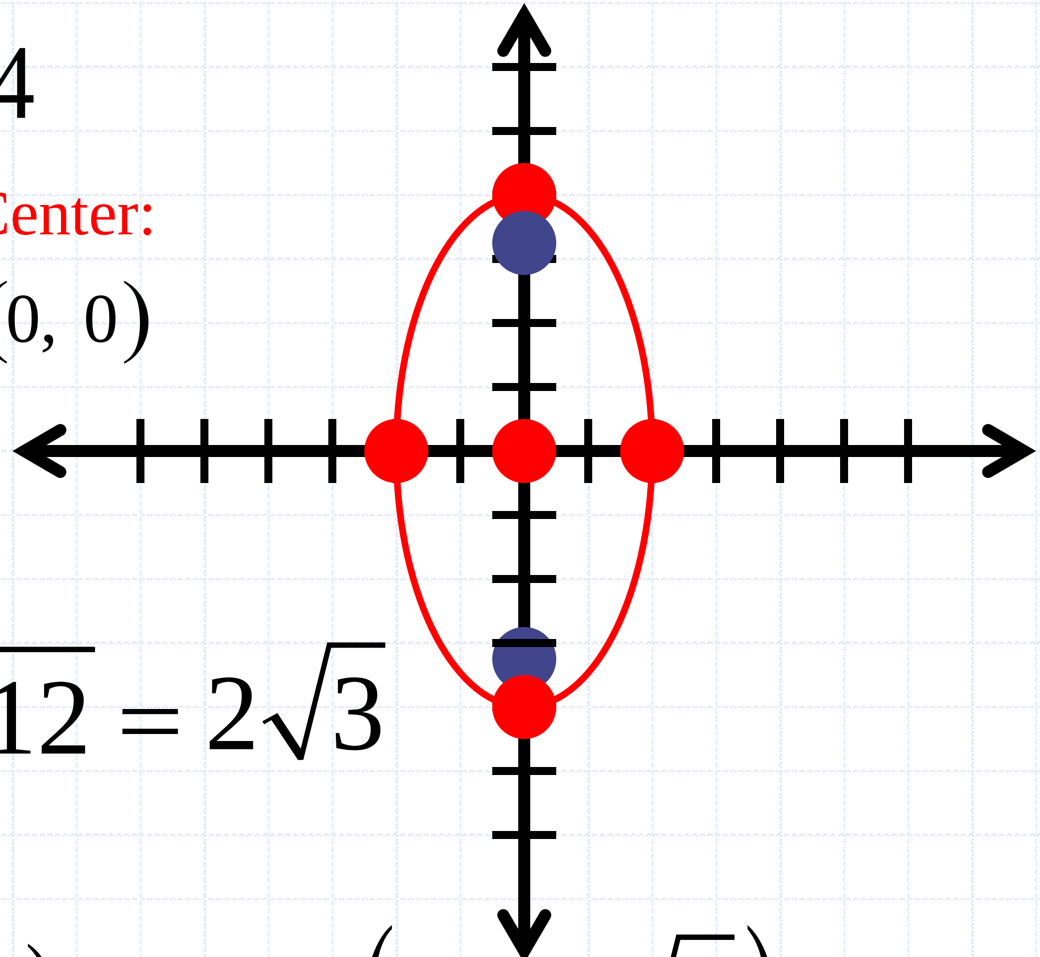
#2 Graph the ellipse

Label the Center, Vertices, Co-Verticies, and Foci.

$$16x^2 + 4y^2 = 64$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

Center: $(0,\ 0)$



$$a = 4 \quad b = 2$$

$$c = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$$

Vertices $\left(0, \pm 4\right)$

Co-Vertices $\left(\pm 2,\ 0\right)$ Focus $\left(0, \pm 2\sqrt{3}\right)$

#3 Graph the ellipse

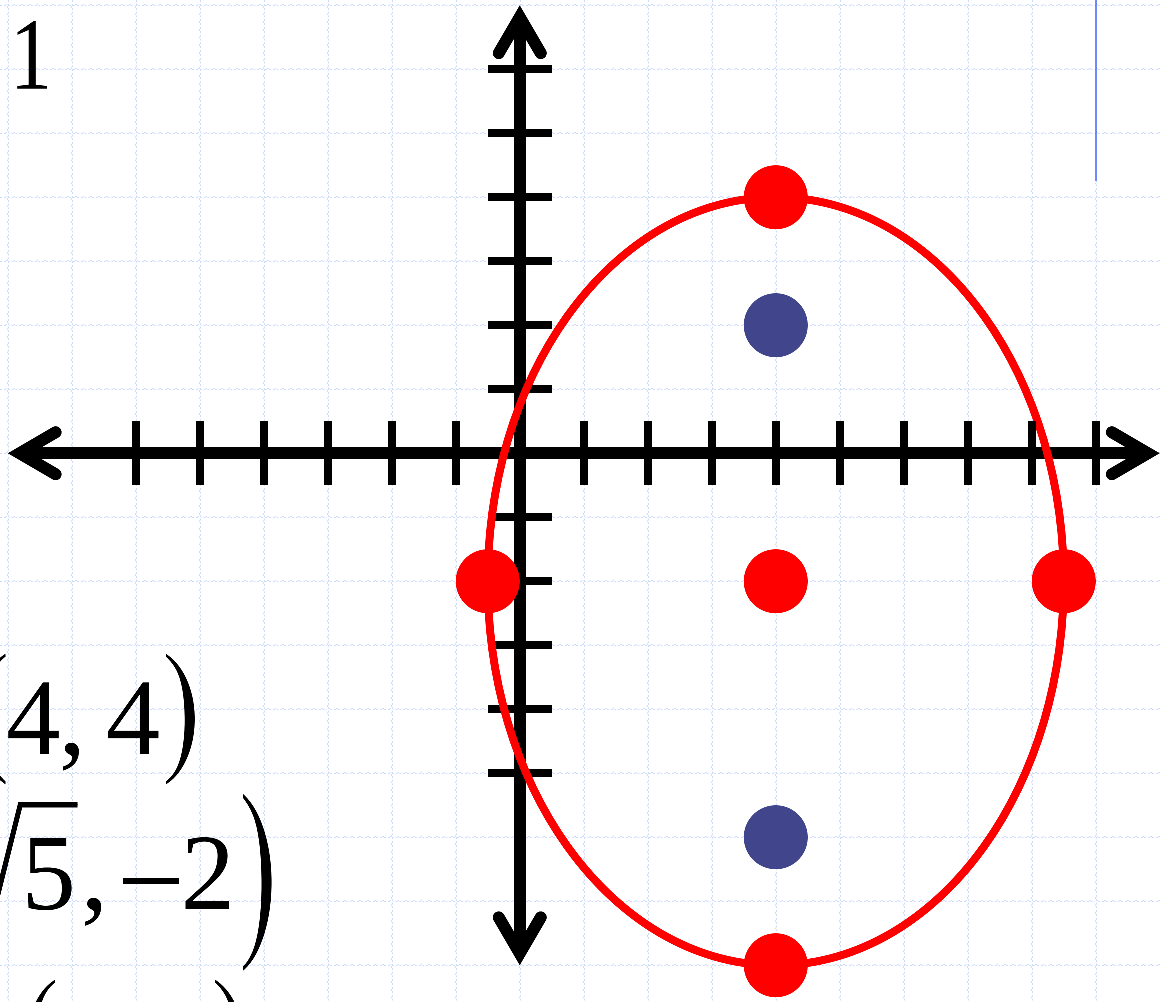
Label the Center Vertices, Co-Verticies, and Foci.

$$\frac{(x-4)^2}{20}+\frac{(y+2)^2}{36}=1$$

$a=6$ $b=2\sqrt{5}$

$c=\sqrt{36-20}$

$c=4$



Center $(4,-2)$

Vertices $(4,-8),(4,4)$

Co-vertices $\left(4\pm 2\sqrt{5},-2\right)$

Foci $(4,-6),(4,2)$

#4 Draw the ellipse

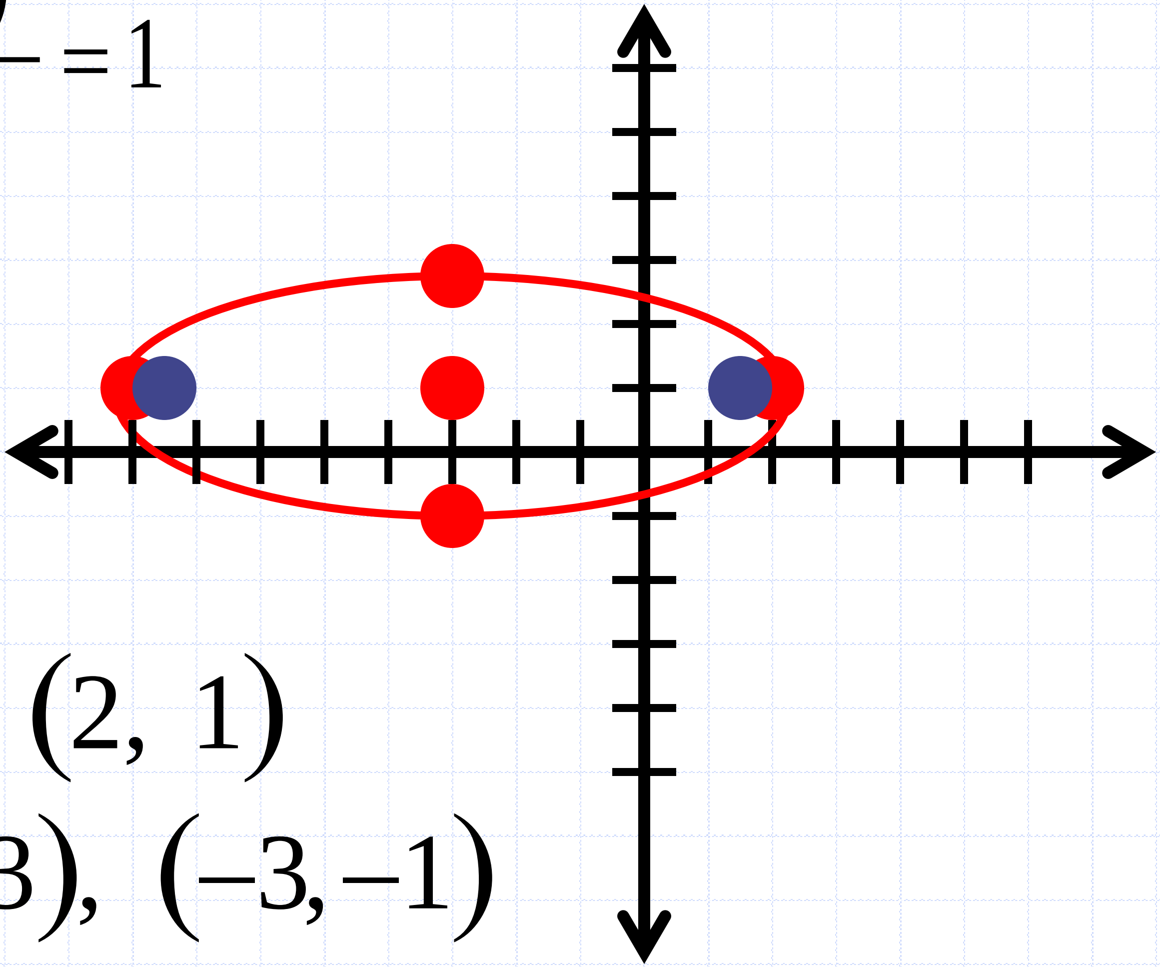
Label the Center, Vertices, Co-Verticies, and Foci.

$$\frac{(x+3)^2}{25} + \frac{(y-1)^2}{4} = 1$$

$a = 5 \quad b = 2$

$$c = \sqrt{25-4}$$

$$c = \sqrt{21} \approx 4.6$$



Center $(-3, 1)$

Vertices $(-8, 1), \ (2, 1)$

Co-vertices $(-3, 3), \ (-3, -1)$

Foci $\left(-3 \pm \sqrt{21}, 1\right)$

#5 Writing Equations of Ellipses

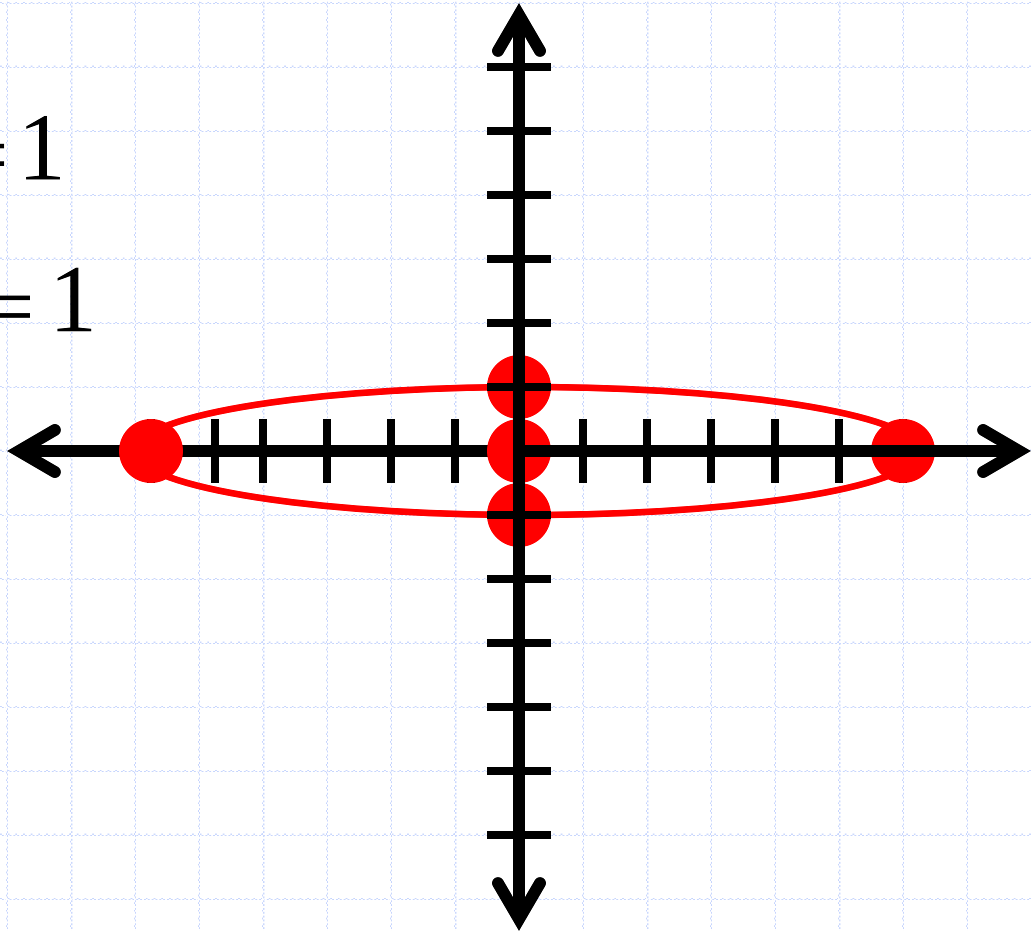
- Write an equation of the ellipse with the Vertex (-6, 0), Co-Vertex (0, -1), and Center (0, 0).

Horizontal:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$h=0 \quad k=0 \quad a=6 \quad b=1$

$$\frac{x^2}{36}+\frac{y^2}{1}=1$$



#6 Writing Equations of Ellipses

- Write an equation of the ellipse with the Center (1, 4). Focus $\left(1, 4+\sqrt{12}\right)$, and Vertex (1,0),

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

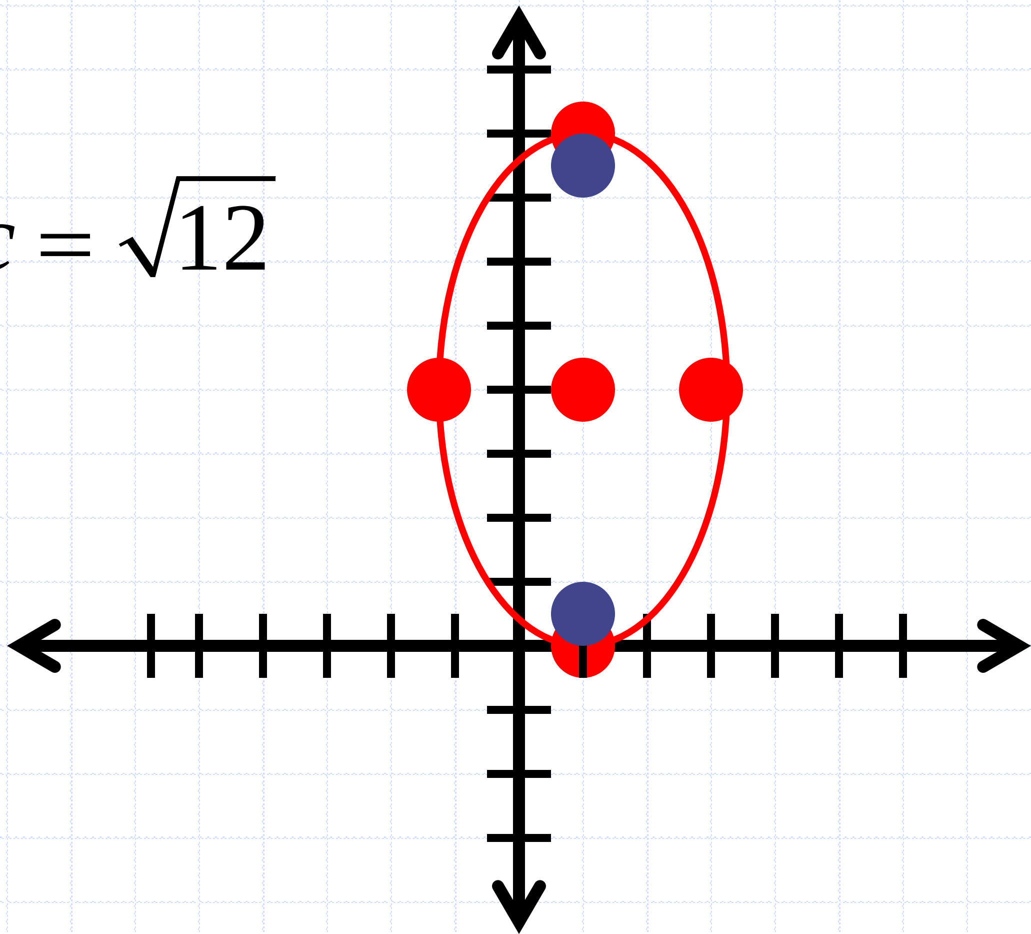
$$h=1 \quad k=4 \quad a=4 \quad c=\sqrt{12}$$

$$\left(\sqrt{12}\right)^2=4^2-b^2$$

$$b^2=16-12$$

$$b=2$$

$$\frac{(x-1)^2}{4}+\frac{(y-4)^2}{16}=1$$



#7 Writing Equations of Ellipses

- Write an equation of the ellipse with the Vertex (-1,-2), Focus (-1, -1), and Center (-1, 3).

Vertical:

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

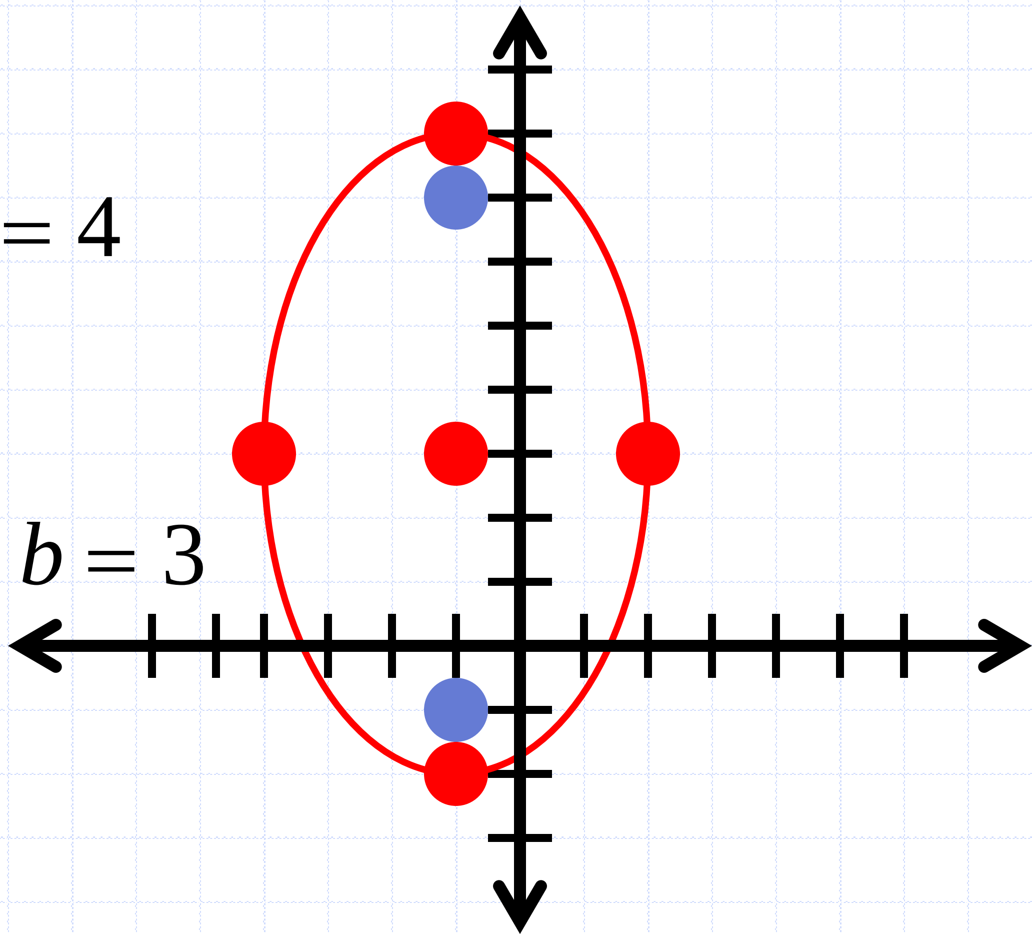
$h=-1 \quad k=3 \quad a=5 \quad c=4$

$c^2=a^2-b^2$

$4^2=5^2-b^2$

$b^2=25-16 \qquad b^2=9 \qquad b=3$

$$\frac{(x+1)^2}{9}+\frac{(y-3)^2}{25}=1$$



#8 Write the equation of the Ellipse in standard form.

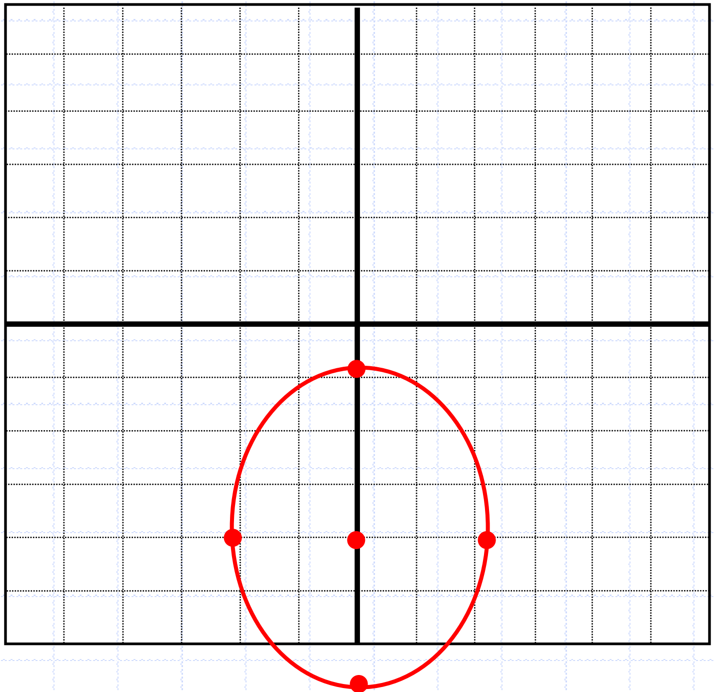
$$2x^2 + y^2 + 8y + 6 = 0$$

$$2x^2 + y^2 + 8y = -6$$

$$2x^2 + \left(y^2 + 8y + \underline{16}\right) = -6 + \underline{16}$$

$$2x^2 + \left(y + 4\right)^2 = 10$$

$$\frac{x^2}{5} + \frac{\left(y + 4\right)^2}{10} = 1$$



#9 Write the equation of the Ellipse in standard form.

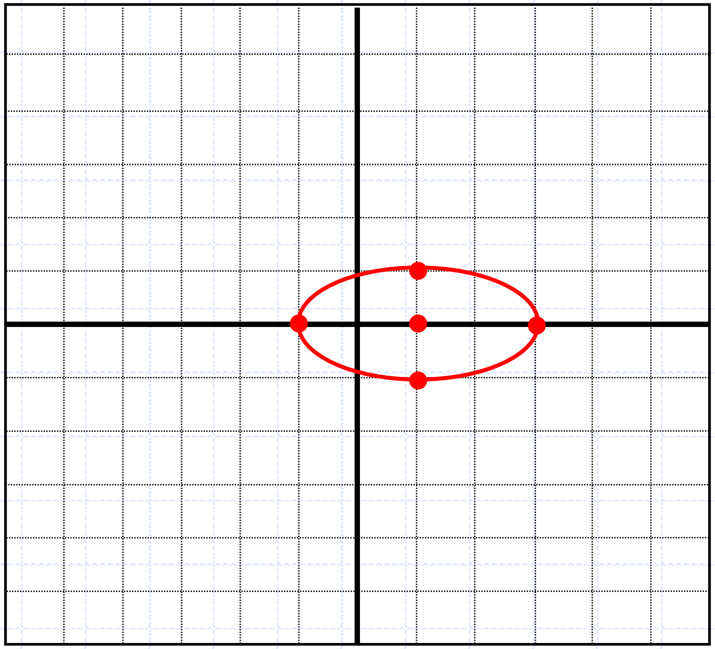
$$x^2 + 4y^2 - 2x - 3 = 0$$

$$x^2 - 2x + 4y^2 = 3$$

$$x^2 - 2x + \underline{\ 1\ } + 4y^2 = 3 + \underline{\ 1\ }$$

$$(x-1)^2 + 4y^2 = 4$$

$$\frac{(x-1)^2}{4} + \frac{y^2}{1} = 1$$



9.2
Hyperbolas

Standard Forms of a Hyperbola

- The standard form of the equation of an hyperbola with center at (*h*, *k*).

Horizontal

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

Vertices $\left(h\pm a,k\right)$

Asymptotes $y=\pm\frac{b}{a}x$

Vertical

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Vertices $\left(h,k\pm a\right)$

Asymptotes $y=\pm\frac{a}{b}x$

$$Foci:\ c^2=a^2+b^2$$

Foci $\left(h\pm c,k\right)$

Foci $\left(h,k\pm c\right)$

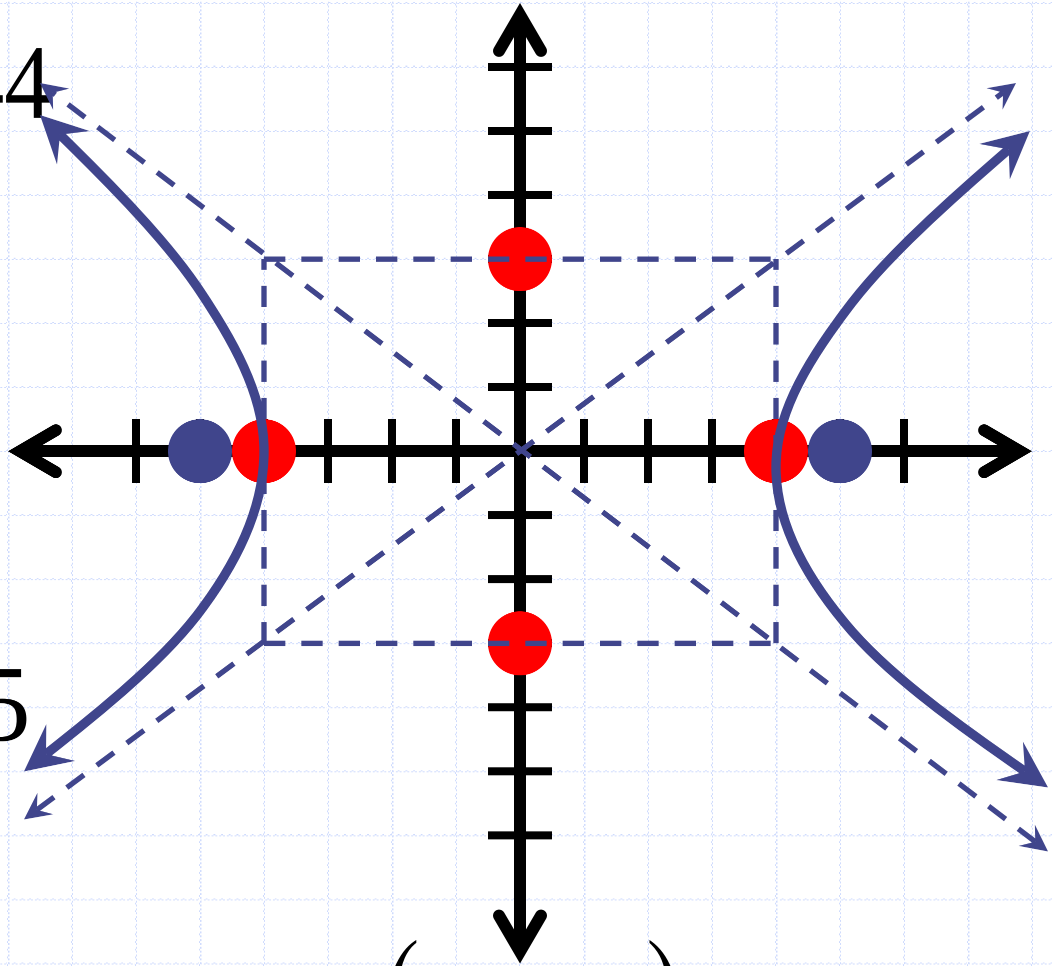
#1 Graph the hyperbola

$$9x^2 - 16y^2 = 144$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$a = 4 \quad b = 3$$

$$c = \sqrt{16+9} = 5$$



Vertices $(\pm 4, 0)$

Asymptotes $y = \pm \frac{3}{4}x$ Focus $(\pm 5, 0)$

#2 Graph the hyperbola

$$y^2 - 25x^2 = 25$$

$$\frac{y^2}{25} - x^2 = 1$$

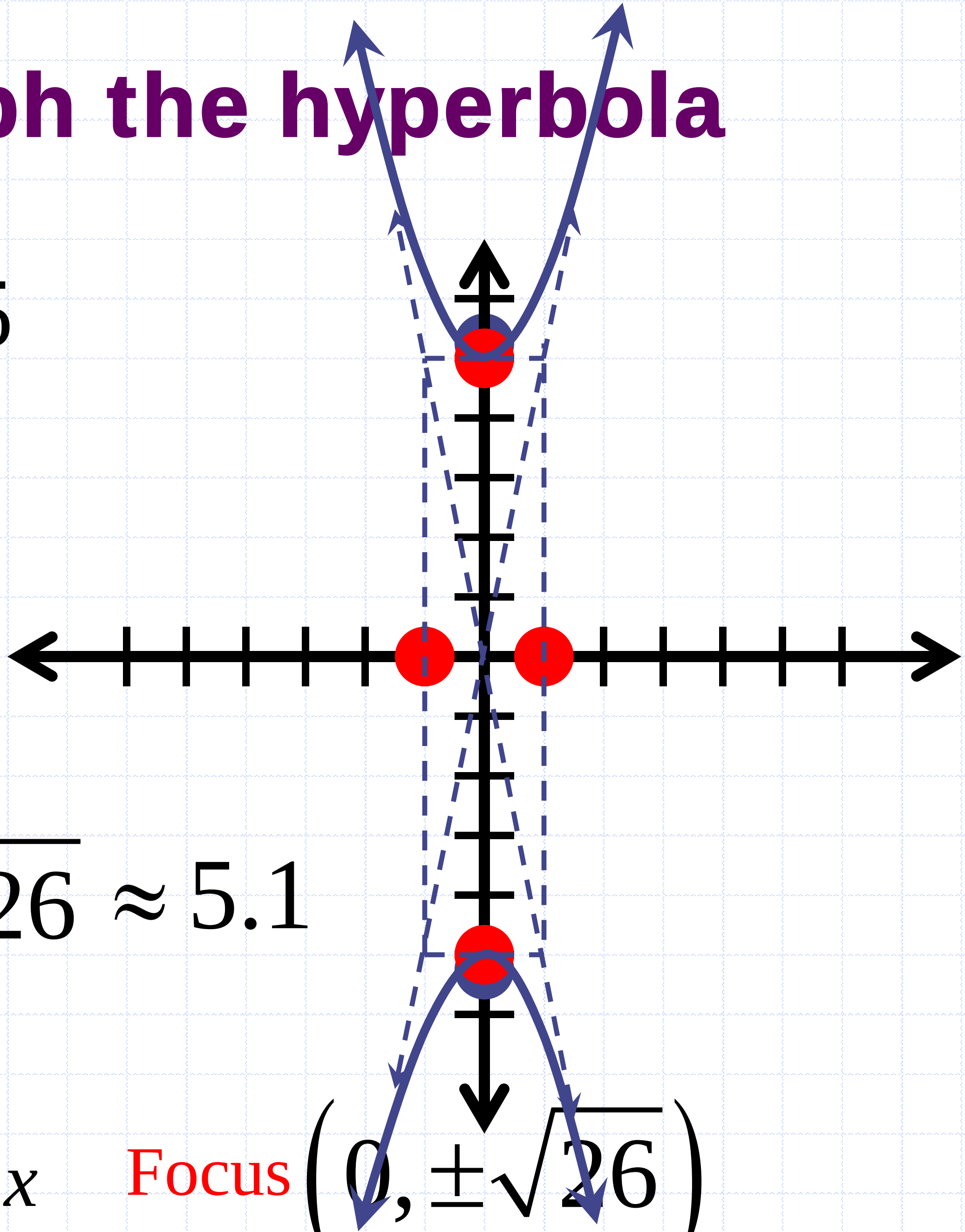
$$a = 5 \quad b = 1$$

$$c = \sqrt{25+1} = \sqrt{26} \approx 5.1$$

Vertices $(0, \pm 5)$

Asymptotes $y = \pm \frac{5}{1} x$

Focus $\left(0, \pm\sqrt{26}\right)$



#3 Graph the hyperbola

$$\frac{(x-2)^2}{4}-\frac{(y+2)^2}{16}=1$$

Center $(2,-2)$

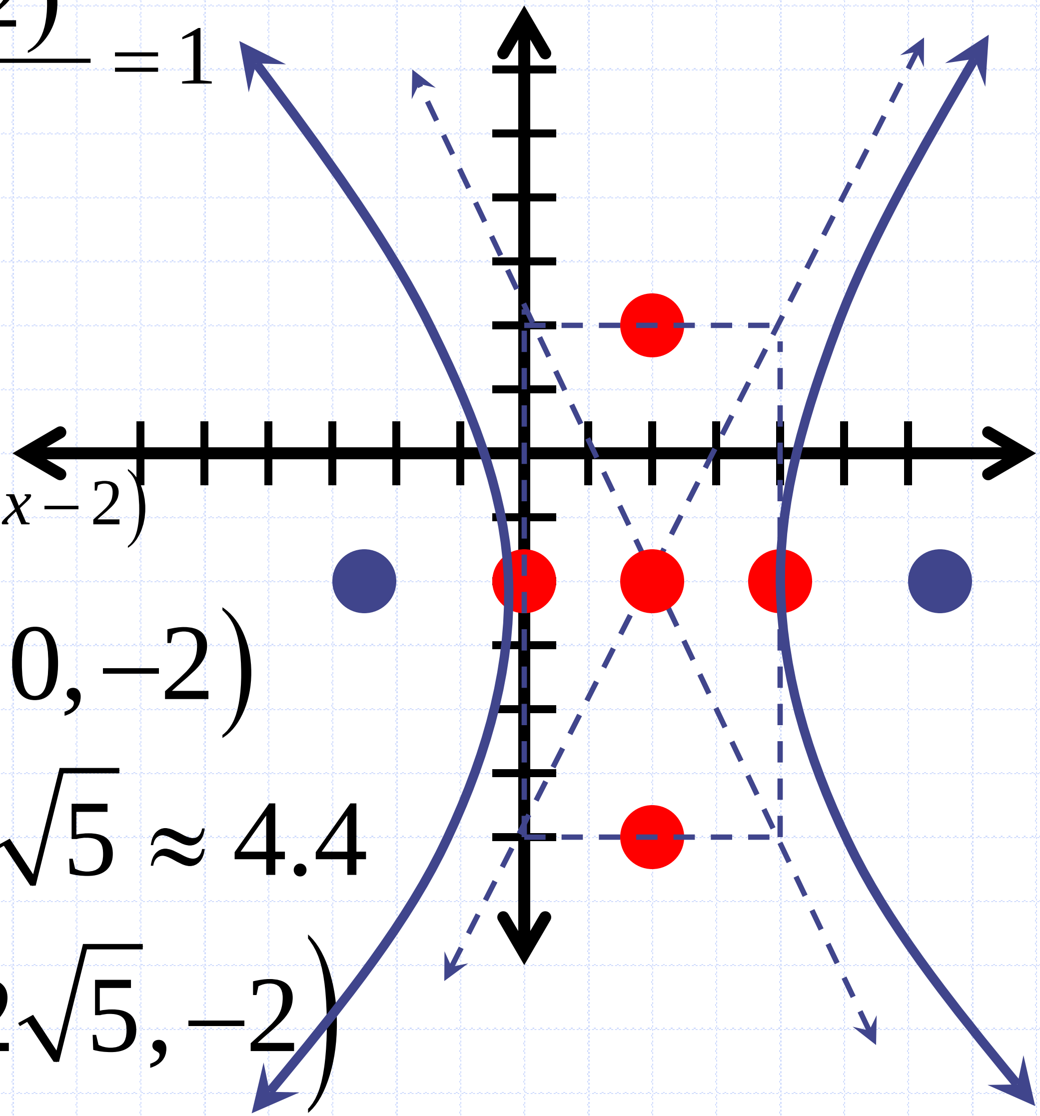
$$a=2 \quad b=4$$

Asymptotes $(y+2)=\pm 2(x-2)$

Vertices $(4,-2),(0,-2)$

$$c=\sqrt{16+4}=2\sqrt{5}\approx 4.4$$

Focus $\left(2\pm 2\sqrt{5},-2\right)$

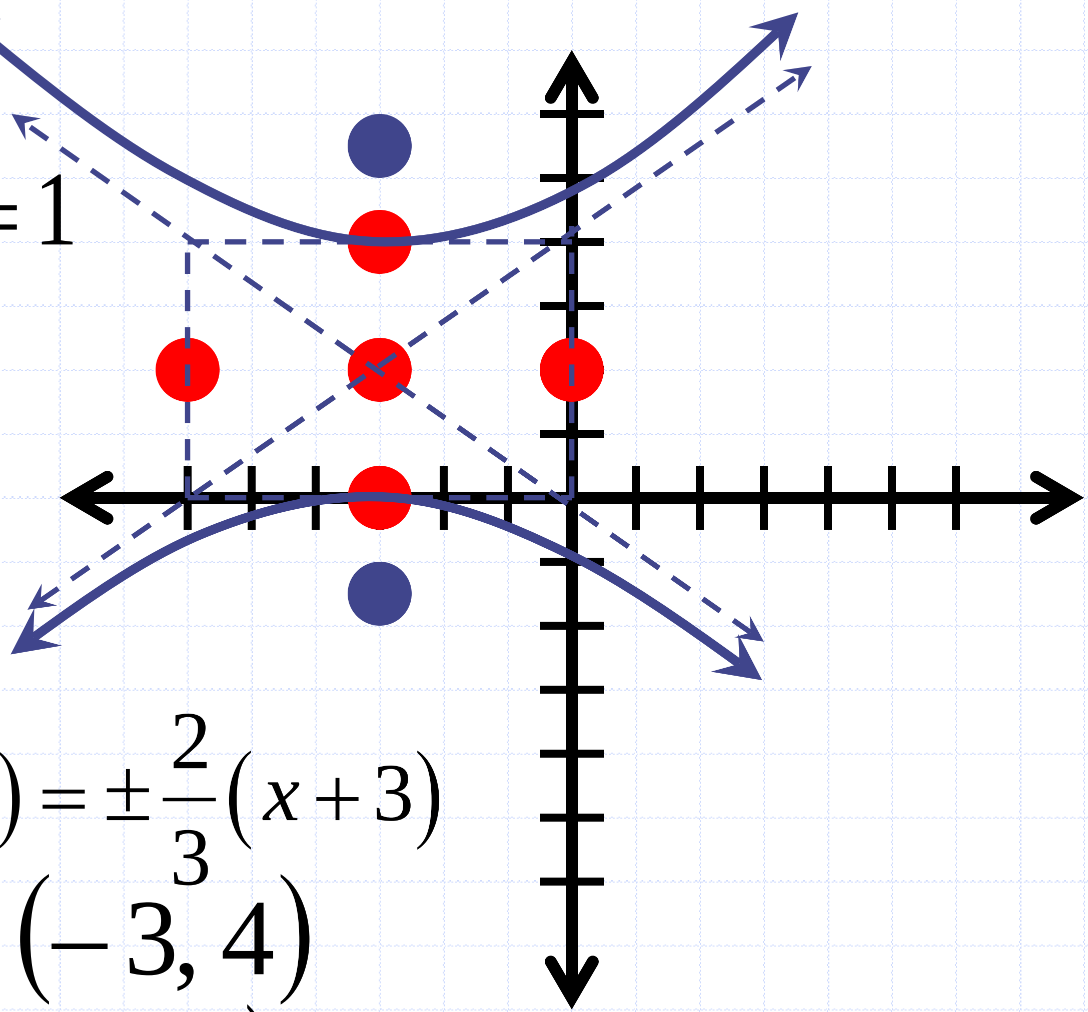


#4 Graph the hyperbola

$$\frac{(y-2)^2}{4} - \frac{(x+3)^2}{9} = 1$$

$a = 2 \quad b = 3$

$c \approx 3.6$



Center $(-3, 2)$

Asymptotes $(y-2) = \pm\frac{2}{3}(x+3)$

Vertices $(-3, 0), (-3, 4)$

Foci $\left(-3, 2 \pm \sqrt{13}\right)$

#5 Writing Equations of Hyperbolas

- Write an equation of the hyperbola with foci at (0, ±5) and vertices at (0, ±3)

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

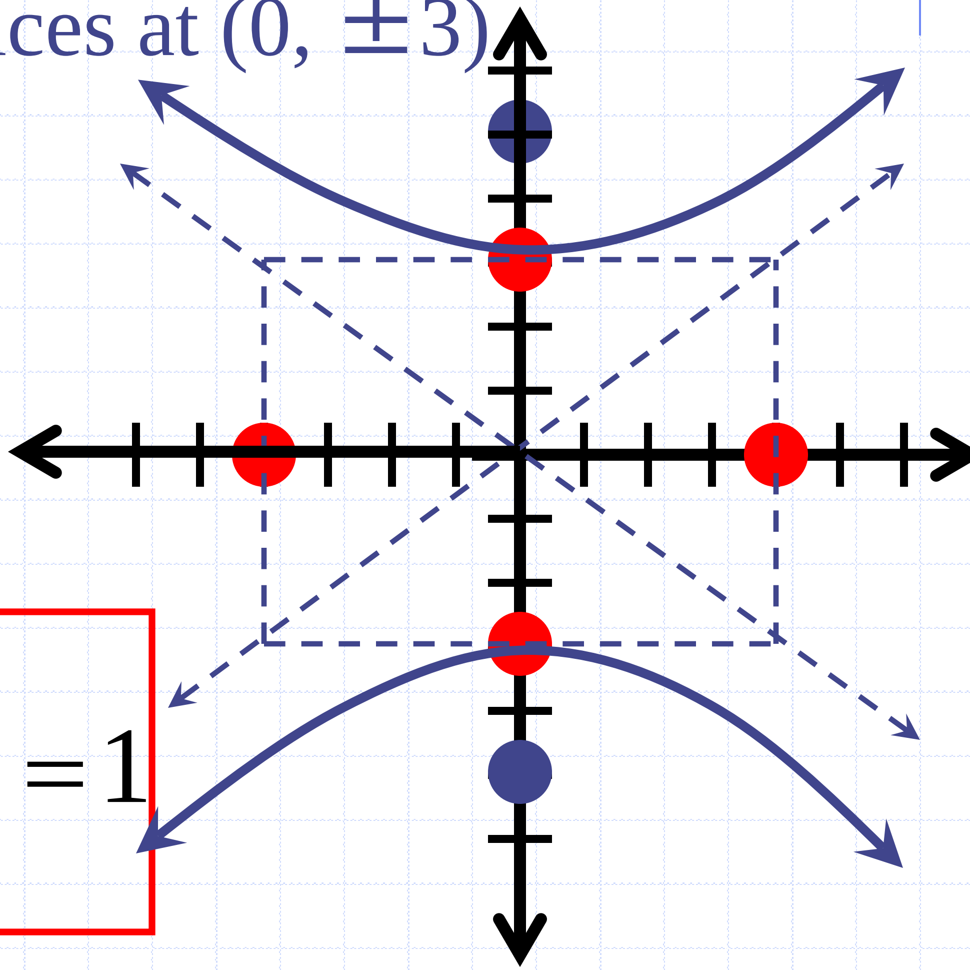
$$a = 3 \quad c = 5$$

$$c^2 = a^2 + b^2$$

$$25 = 9 + b^2$$

$$b = 4$$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$



#6 Writing Equations of Hyperbolas

- Write an equation of the hyperbola with foci at $\left(3 \pm \sqrt{10}, 3\right)$ and vertices at (4,3) and (2,3).

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center: $(3,3)$

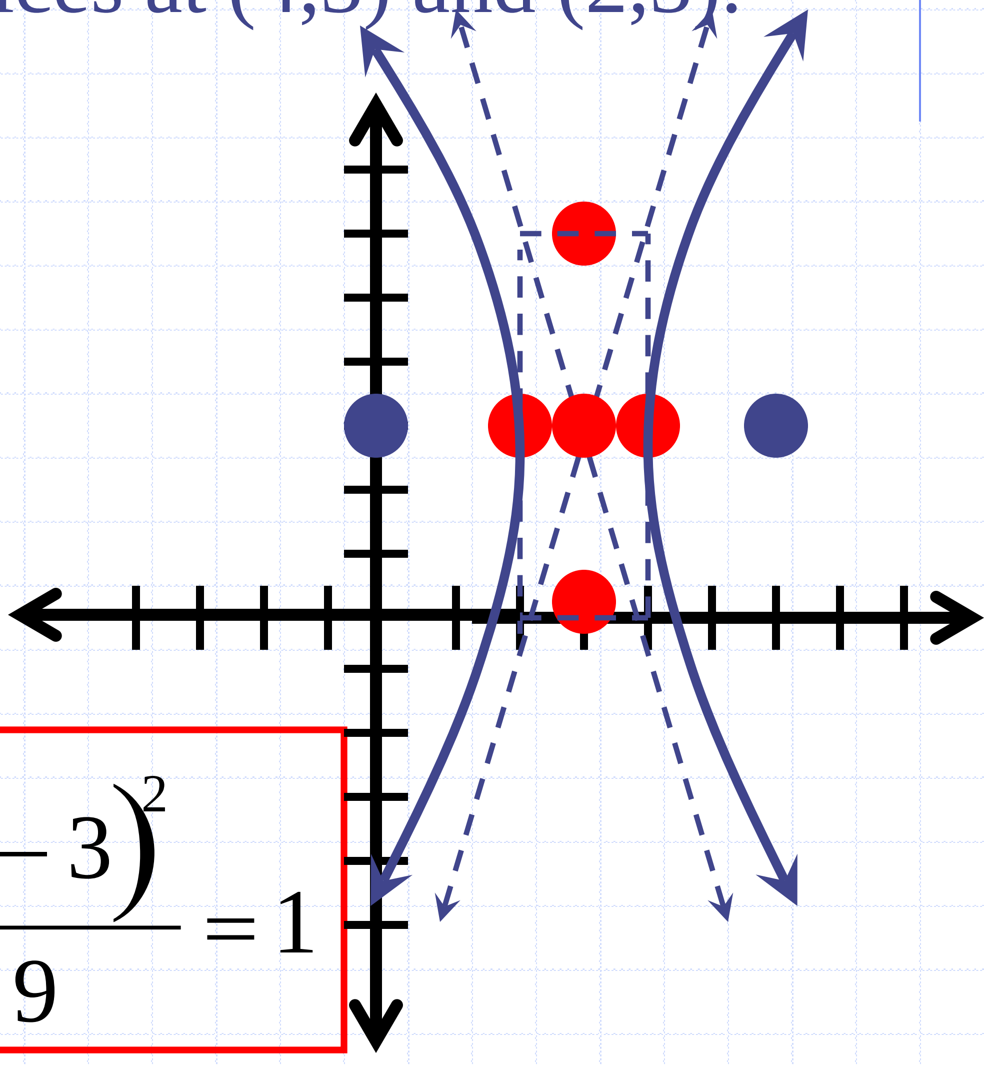
$a = 1 \quad c = \sqrt{10}$

$c^2 = a^2 + b^2$

$10 = 1 + b^2$

$b = 3$

$$\frac{(x-3)^2}{1} - \frac{(y-3)^2}{9} = 1$$



#7 Writing Equations of Hyperbolas

- Write an equation of the hyperbola with foci at (4,5) and (4,-3) and vertices at (4,4) and (4,-2).

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Center: $(4,1)$

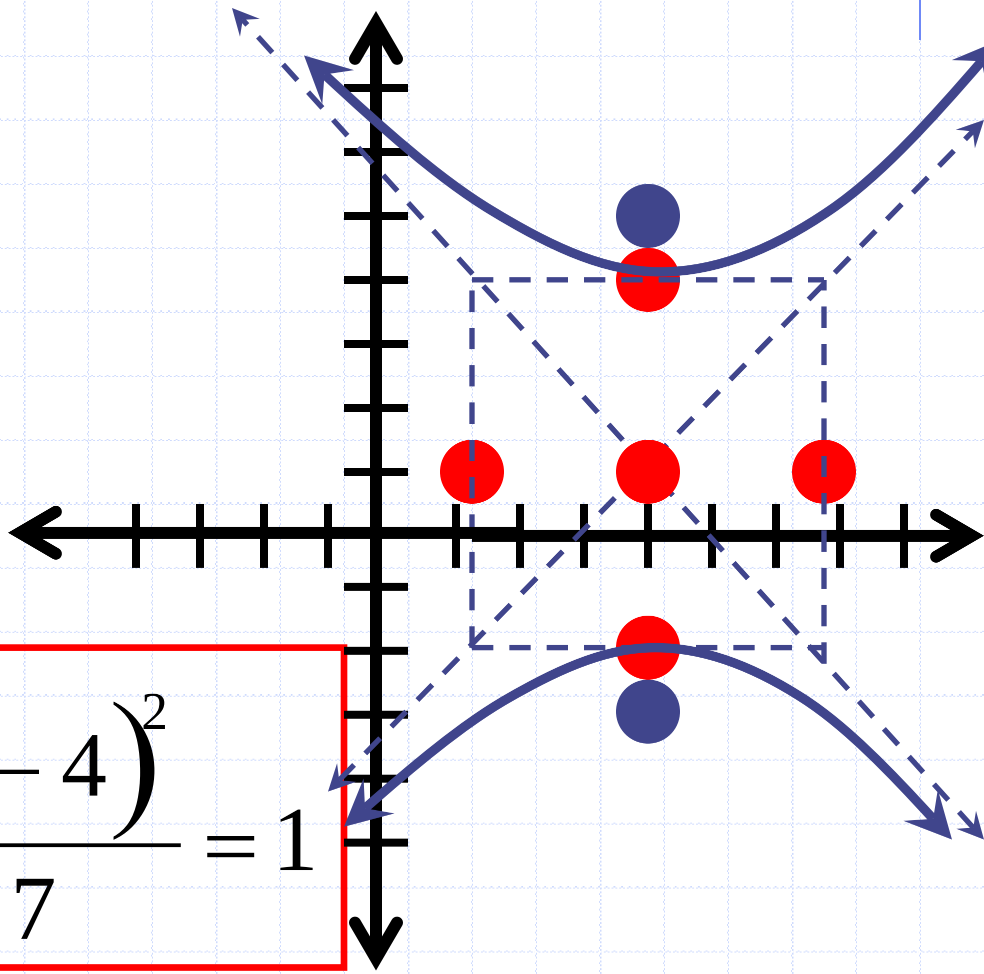
$a=3$ $\quad c=4$

$c^2=a^2+b^2$

$16=9+b^2$

$b=\sqrt{7}$

$$\frac{(y-1)^2}{9}-\frac{(x-4)^2}{7}=1$$



#8 Write in Standard Form

$$4x^2 - y^2 - 16x - 4y - 4 = 0$$

$$4x^2 - 16x - y^2 - 4y = 4$$

$$4\left(x^2 - 4x + \underline{\ 4\ }\right) - \left(y^2 + 4y + \underline{\ 4\ }\right) = 4 + 4\left(\underline{\ 4\ }\right) - 1\left(\underline{\ 4\ }\right)$$

$$4(x-2)^2 - (y+2)^2 = 4 + 16 - 4$$

$$4(x-2)^2 - (y+2)^2 = 16$$

$$\frac{(x-2)^2}{4} - \frac{(y+2)^2}{16} = 1$$

#9 Write in Standard Form

$$-9x^2 + 16y^2 + 54x + 64y - 161 = 0$$

$$16y^2 + 64y - 9x^2 + 54x = 161$$

$$16\left(y^2 + 4y + \underline{\ 4\ }\right) - 9\left(x^2 - 6x + \underline{\ 9\ }\right) = 161 + 16(\underline{\ 4\ }) - 9(\underline{\ 9\ })$$

$$16(y+2)^2 - 9(x-3)^2 = 161 + 64 - 81$$

$$16(y+2)^2 - 9(x-3)^2 = 144$$

$$\boxed{\frac{(y+2)^2}{9} - \frac{(x-3)^2}{16} = 1}$$