

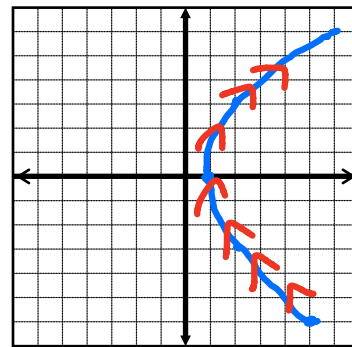
### Parametric Equations (9.5)

<p><b>Plane Curves and Parametric Equations</b>                  Suppose that <math>t</math> is a number in an interval <math>I</math>. A plane curve is the set of ordered pairs <math>(x, y)</math>, where <math>x = f(t)</math>, <math>y = g(t)</math> for <math>t</math> in interval <math>I</math>.                  The variable <math>t</math> is called a <u>parameter</u>, and the equations <math>x = f(t)</math> and <math>y = g(t)</math> are called <u>parametric equations</u>.</p>	<p><b>Graphing a Plane Curve Described by Parametric Equations</b></p> <ol style="list-style-type: none"> <li>1. Select values of <math>t</math> on the given interval.</li> <li>2. For each value of <math>t</math>, use the given parametric equations to compute <math>x</math> and <math>y</math>.</li> <li>3. Plot the points <math>(x, y)</math> in the order of increasing <math>t</math> and connect them with a smooth curve.</li> </ol>
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**Graphing a Curve Defined by Parametric Equations:** Graph the plane curves defined by the parametric equations.

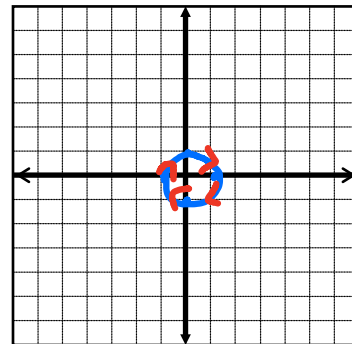
1)  $x = t^2 + 1, \quad y = 3t, \quad -2 \leq t \leq 2$

$t$	$x$	$y$	$(x, y)$
-2	5	-6	(5, -6)
-1	2	-3	(2, -3)
0	1	0	(1, 0)
1	2	3	(2, 3)
2	5	6	(5, 6)



2)  $x = -\sin t, \quad y = -\cos t, \quad 0 \leq t < 2\pi$

$t$	$x$	$y$	$(x, y)$
0	0	-1	(0, -1)
$\pi/2$	-1	0	(-1, 0)
$\pi$	0	1	(0, 1)
$3\pi/2$	1	0	(1, 0)
$2\pi$	0	-1	(0, -1)

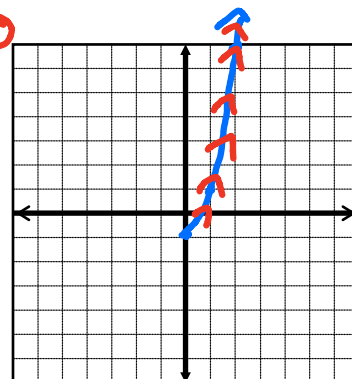


**Finding and Graphing the Rectangular Equation of a Curve Defined Parametrically:** Eliminate the parameter  $t$ . Then use the rectangular equation to sketch the plane curve represented by the given parametric equations. Use arrows to show the orientation of the curve corresponding to increasing values of  $t$ . (If an interval for  $t$  is not specified, assume that  $-\infty < t < \infty$ ).

3)  $x = \sqrt{t}, \quad y = 2t - 1$

$x^2 = t$ 
 $y = 2x^2 - 1$ 
 $t \geq 0$   
 $y + 1 = 2x^2$

$x^2 = \frac{1}{2}(y + 1)$



$t$	$(x, y)$
0	(0, -1)
1	(1, 1)

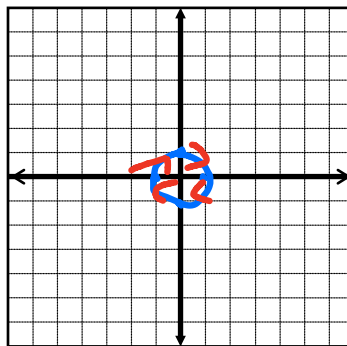
4)  $x = -\sin t, y = -\cos t, 0 \leq t < 2\pi$

$x^2 = \sin^2 t, y^2 = \cos^2 t$

$x^2 + y^2 = \sin^2 t + \cos^2 t$

$x^2 + y^2 = 1$  Circle

$t$	$(x, y)$
0	(0, -1)
$\pi/2$	(-1, 0)



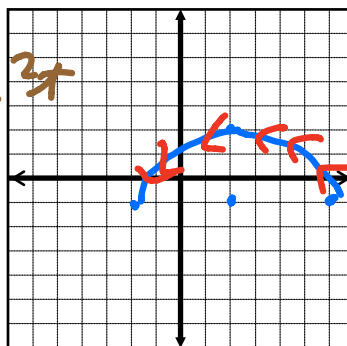
5)  $x = 2 + 4\cos t, y = -1 + 3\sin t, 0 \leq t \leq \pi$

$\left(\frac{x-2}{4}\right)^2 = \cos^2 t, \left(\frac{y+1}{3}\right)^2 = \sin^2 t$

$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = \cos^2 t + \sin^2 t$

$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1$  Ellipse

$t$	$(x, y)$
0	(6, -1)
$\pi/2$	(2, 2)
$\pi$	(-2, -1)



6)  $x = t^2, y = t - 2$

$x = t^2, y + 2 = t$

$x = (y + 2)^2$  parabola

$(y + 2)^2 = x$

$t$	$(x, y)$
0	(0, -2)
1	(1, -1)

