

## 8.1, 8.2 (Matrix Solutions to Linear Systems)

## Gaussian Elimination

$$\text{Row-Echelon Form } \left[ \begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array} \right] \longrightarrow \text{Back-Order Substitution } \begin{array}{l} x + ay + bz = c \\ y + dz = e \\ z = f \end{array}$$

## Gauss-Jordan Elimination

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right] \longrightarrow \begin{array}{l} x = a \\ y = b \\ z = c \end{array}$$

**Back-Order Substitution:** Write the linear system and solve using back-order substitution.

$$1) \left[ \begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 3/2 & -2 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$2) \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

**Matrix Operations:** Perform the matrix operation and write the new matrix.

$$3) \left[ \begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array} \right] -3R_1 + R_2$$

$$4) \left[ \begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array} \right] R_1 - R_3$$

**Solving Systems Using Matrices:** *Use Gaussian elimination with back-substitution or Gauss-Jordan elimination.*

$$3a + b - c = 0$$

5)  $2a + 3b - 5c = 1$

$$a - 2b + 3c = -4$$

$$2x + y = z + 1$$

6)  $2x = 1 + 3y - z$

$$x + y + z = 4$$

## 8.2 (Inconsistent and Dependent Systems and Their Applications)

### System with No Solutions

$$\text{Row-Echelon Form } \left[ \begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 0 & f \end{array} \right] \longrightarrow 0 \neq f$$

### System with Infinite Many Solutions

$$\left[ \begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 0 & 0 \end{array} \right] \longrightarrow 0 = 0$$

**Solving Using Gaussian Elimination:** Use Gaussian elimination to find the complete solution to each system of equations, or show that none exist.

$$x - 2y - z = -5$$

7)  $2x - 3y - z = 0$

$$3x - 4y - z = 1$$

$$x - 2y - z = 5$$

8)  $2x - 5y + 3z = 6$

$$x - 3y + 4z = 1$$