8.1
Matrix Solutions to Linear Systems

- Using Matrix Row Operations
- Solving by Using Gaussian Elimination with back-substitution
- Using Gauss-Jordan Elimination

Back-Substitution

1. $$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 3/2 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$x + y = 3$$

$$y + (3/2)z = -2$$

$$z = 0$$

$$\boxed{(5, -2, 0)}$$

Back-Substitution

2.

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$a + 2b - c = 2$

$b + c - 2d = -3$

$c - d = -2$

$d = 3$

$$\boxed{(-1, 2, 1, 3)}$$

Matrix Operations

3. $\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right] \quad -3R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ -3(1)+3 & -3(-1)+3 & -3(5)-1 & -3(-6)+10 \\ 1 & 3 & 2 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 6 & -16 & 28 \\ 1 & 3 & 2 & 5 \end{array}\right]$$

Matrix Operations

4. $\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right] R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1-1 & -1-3 & 5-2 & -6-5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 0 & -4 & 3 & -11 \end{array}\right]$$

Solving Systems Using Matrices

5. $3a + b - c = 0$

$2a + 3b - 5c = 1$

$a - 2b + 3c = -4$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 0 \\ 2 & 3 & -5 & 1 \\ 1 & -2 & 3 & -4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 2 & 3 & -5 & 1 \\ 3 & 1 & -1 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 7 & -11 & 9 \\ 3 & 1 & -1 & 0 \end{array}\right] \quad -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 7 & -11 & 9 \\ 0 & 7 & -10 & 12 \end{array}\right] -3R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 7 & -11 & 9 \\ 0 & 0 & -1 & -3 \end{array}\right] R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 1 & -11/7 & 9/7 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \frac{1}{7}R_2, -R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 1 & -11/7 & 9/7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$a - 2b + 3c = -4$$

$$b - (11/7)c = 9/7$$

$$c = 3$$

$$\boxed{(-1, 6, 3)}$$

Solving Systems Using Matrices

6. $2x + y = z + 1$

$2x = 1 + 3y - z$

$x + y + z = 4$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 2 & -3 & 1 & 1 \\ 1 & 1 & 1 & 4 \end{array}\right]$$

$$\boxed{(1,1,2)}$$

8.2
Independent and Dependent Systems

- Recognizing the differences between independent and dependent systems
- Solving Using Matrices to find independent and dependent systems

Solving Systems Using Matrices

7. $x - 2y - z = -5$

$2x - 3y - z = 0$

$3x - 4y - z = 1$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 2 & -3 & -1 & 0 \\ 3 & -4 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 1 & 10 \\ 3 & -4 & -1 & 1 \end{array}\right] \quad -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 2 & 16 \end{array}\right] \quad -3R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 1 & 10 \\ 0 & 0 & 0 & -4 \end{array}\right] \quad -2R_2 + R_3$$

No Solutions; $\varnothing$; Empty Set

$$0 \neq -4$$

Independent Solution

Solving Systems Using Matrices

8. $x - 2y - z = 5$

$2x - 5y + 3z = 6$

$x - 3y + 4z = 1$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 2 & -5 & 3 & 6 \\ 1 & -3 & 4 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & -1 & 5 & -4 \\ 1 & -3 & 4 & 1 \end{array}\right] \quad -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & -1 & 5 & -4 \\ 0 & -1 & 5 & -4 \end{array}\right] \quad -R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 + R_3, \text{-}R_2$$

$$\left(\_\_\_\_, \_\_\_\_, z\right)$$

$$x - 2y - z = 5$$

$$y - 5z = 4 \longrightarrow y = 5z + 4$$

$$\left(\_\_\_\_\_\_, 5z + 4, z\right)$$

$$x - 2\left(5z + 4\right) - z = 5$$

$$x = 11z + 13$$

$$\boxed{\left(11z + 13, 5z + 4, z\right)}$$