

8.5 (Determinants and Cramer's Rule)

Determinant of a 2x2 Matrix

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}(a_{22}) - a_{12}(a_{21})$$

Determinant of a 3x3 Matrix

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

Evaluating the Determinant: Evaluate each determinant.

1) $\begin{vmatrix} 7 & 9 \\ -2 & -5 \end{vmatrix}$

2) $\begin{vmatrix} 4 & 0 & 0 \\ 3 & -1 & 4 \\ 2 & -3 & 5 \end{vmatrix}$

Solving a Two Variable System Using Cramer's Rule

If $\begin{matrix} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{matrix}$, Then $x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ and $y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$, where $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$

Solving a Two Variable System Using Cramer's Rule: Use Cramer's rule to solve each system or to determine that the system is inconsistent or contains dependent equations.

3) $\begin{matrix} 3x + 2y = 2 \\ 2x + 2y = 3 \end{matrix}$

4) $\begin{matrix} 3x = 2 - 3y \\ 2y = 3 - 2x \end{matrix}$

Solving a Three Variable System Using Cramer's Rule

$$\begin{aligned} & a_1x + b_1y + c_1z = d_1 \\ \text{If } & a_2x + b_2y + c_2z = d_2, \quad \text{Then } x = \frac{D_x}{D}, y = \frac{D_y}{D} \quad \text{and } z = \frac{D_z}{D} \\ & a_3x + b_3y + c_3z = d_3 \end{aligned}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Solving a Three Variable System Using Cramer's Rule: Use Cramer's rule to solve each system or to determine that the system is inconsistent or contains dependent equations.

$$4x - 5y - 6z = -1$$

$$2x + 2y + 3z = 10$$

5) $x - 2y - 5z = -12$

6) $4x - y + z = -5$

$$2x - y = 7$$

$$5x - 2y + 6z = 1$$