8.5
Determinants and Cramer's Rule

Evaluating the Determinant

1. $\begin{vmatrix} 7 & 9 \\ -2 & -5 \end{vmatrix}$

$$D = -35 + 18 = \boxed{-17}$$

Evaluating the Determinant

2. $\begin{vmatrix} 4 & 0 & 0 \\ 3 & -1 & 4 \\ 2 & -3 & 5 \end{vmatrix}$

$$4\begin{vmatrix} -1 & 4 \\ -3 & 5 \end{vmatrix} - 3\begin{vmatrix} 0 & 0 \\ -3 & 5 \end{vmatrix} + 2\begin{vmatrix} 0 & 0 \\ -1 & 4 \end{vmatrix}$$

$$D = 4(-5+12) - 3(0-0) + 2(0-0)$$

$$D = 4(7) = 28$$

Solving a Two Variable System

3. $3x+2y=2$

$2x+2y=3$

$$x=\frac{\begin{vmatrix}2 & 2\\ 3 & 2\end{vmatrix}}{\begin{vmatrix}3 & 2\\ 2 & 2\end{vmatrix}}=\frac{4-6}{6-4}=-1 \qquad y=\frac{\begin{vmatrix}3 & 2\\ 2 & 3\end{vmatrix}}{\begin{vmatrix}3 & 2\\ 2 & 2\end{vmatrix}}=\frac{9-4}{6-4}=\frac{5}{2}$$

$$\left(-1,\frac{5}{2}\right)$$

Solving a Two Variable System

4. $3x = 2 - 3y$ $\longrightarrow$ $3x + 3y = 2$

$2y = 3 - 2x$ $\qquad\qquad$ $2x + 2y = 3$

$$x = \frac{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix}} = \frac{4-9}{6-6} = \frac{-5}{0} \qquad y = \frac{\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix}} = \frac{9-4}{6-6} = \frac{5}{0}$$

Inconsistent

Solving a Three Variable System

5. $4x-5y-6z=-1$

$x-2y-5z=-12$

$2x-y=7$

$$D=\begin{vmatrix}4&-5&-6\\1&-2&-5\\2&-1&0\end{vmatrix}, D_x=\begin{vmatrix}-1&-5&-6\\-12&-2&-5\\7&-1&0\end{vmatrix},$$

$$D_y=\begin{vmatrix}4&-1&-6\\1&-12&-5\\2&7&0\end{vmatrix}, D_z=\begin{vmatrix}4&-5&-1\\1&-2&-12\\2&-1&7\end{vmatrix}$$

Solving a Three Variable System

$$D = \begin{vmatrix} 4 & -5 & -6 \\ 1 & -2 & -5 \\ 2 & -1 & 0 \end{vmatrix}$$

$$4\begin{vmatrix} -2 & -5 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} -5 & -6 \\ -1 & 0 \end{vmatrix} + 2\begin{vmatrix} -5 & -6 \\ -2 & -5 \end{vmatrix}$$

$$D = 4(-5) - 1(-6) + 2(13) = 12$$

Solving a Three Variable System

$$D_x = \begin{vmatrix} -1 & -5 & -6 \\ -12 & -2 & -5 \\ 7 & -1 & 0 \end{vmatrix}$$

$$-1\begin{vmatrix} -2 & -5 \\ -1 & 0 \end{vmatrix} + 12\begin{vmatrix} -5 & -6 \\ -1 & 0 \end{vmatrix} + 7\begin{vmatrix} -5 & -6 \\ -2 & -5 \end{vmatrix}$$

$$D_x = -1(-5) + 12(-6) + 7(13) = 24$$

Solving a Three Variable System

$$D_y = \begin{vmatrix} 4 & -1 & -6 \\ 1 & -12 & -5 \\ 2 & 7 & 0 \end{vmatrix}$$

$$4\begin{vmatrix} -12 & -5 \\ 7 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -6 \\ 7 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & -6 \\ -12 & -5 \end{vmatrix}$$

$$D_y = 4(35) - 1(42) + 2(-67) = -36$$

Solving a Three Variable System

$$D_z = \begin{vmatrix} 4 & -5 & -1 \\ 1 & -2 & -12 \\ 2 & -1 & 7 \end{vmatrix}$$

$$4\begin{vmatrix} -2 & -12 \\ -1 & 7 \end{vmatrix} - 1\begin{vmatrix} -5 & -1 \\ -1 & 7 \end{vmatrix} + 2\begin{vmatrix} -5 & -1 \\ -2 & -12 \end{vmatrix}$$

$$D_z = 4(-26) - 1(-36) + 2(58) = 48$$

Solving a Three Variable System

5. $4x-5y-6z=-1$

$x-2y-5z=-12$

$2x-y=7$

$$x=\frac{D_x}{D}, y=\frac{D_y}{D}, z=\frac{D_z}{D}$$

$$x=\frac{24}{12}, y=\frac{-36}{12}, z=\frac{48}{12}$$

$$\boxed{(2,-3,4)}$$

Solving a Three Variable System

6. $2x + 2y + 3z = 10$

$4x - y + z = -5$

$5x - 2y + 6z = 1$

$\boxed{(-1, 3, 2)}$