

HW44: Unit 6 (Limits) Review

1) Complete the table for the function and find the indicate limit

$$1) \lim_{x \rightarrow 2} \frac{3-x}{x-2}$$

x						
$f(x)$						

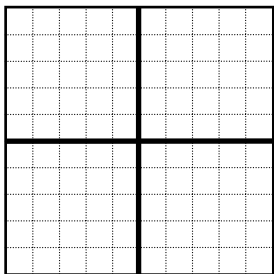
2) Complete the table for the function and find the indicate limit

$$2) \lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2 - x}$$

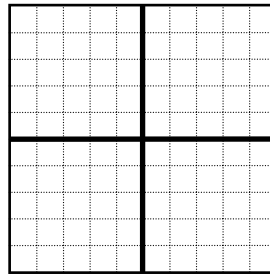
x						
$f(x)$						

Graph the following functions and use your graph to find the indicated limit

$$3) f(x) = \begin{cases} 3x-1, & x < 2 \\ -x^2+1, & x \geq 2 \end{cases} \quad \lim_{x \rightarrow 2} f(x) = \underline{\hspace{2cm}}$$

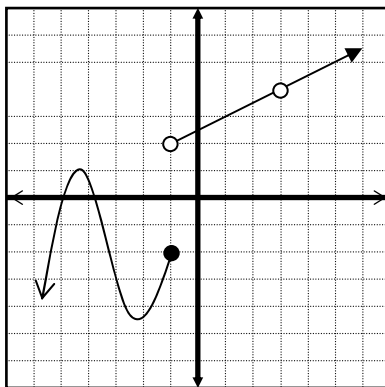


$$4. f(x) = \begin{cases} |x-2|, & x \leq 1 \\ 1, & x > 1 \end{cases} \quad \lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$$



5. Use the graph to the right to find the following.
(Assume that all tick marks are 1 unit apart.)

- a. $\lim_{x \rightarrow 3^-} f(x) = \underline{\hspace{2cm}}$ g. $\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$
- b. $\lim_{x \rightarrow 3^+} f(x) = \underline{\hspace{2cm}}$ h. $\lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}}$
- c. $\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$ i. $\lim_{x \rightarrow -1} f(x) = \underline{\hspace{2cm}}$
- d. $\lim_{x \rightarrow -3^-} f(x) = \underline{\hspace{2cm}}$ j. $f(-1) = \underline{\hspace{2cm}}$
- e. $\lim_{x \rightarrow -3^+} f(x) = \underline{\hspace{2cm}}$ k. $f(3) = \underline{\hspace{2cm}}$
- f. $\lim_{x \rightarrow -3} f(x) = \underline{\hspace{2cm}}$



6. Given $f(x) = \begin{cases} x^2 + 6 & x < 2 \\ x^3 + 2 & x \geq 2 \end{cases}$

- a). $\lim_{x \rightarrow 2^-} f(x) =$
- b). $\lim_{x \rightarrow 2^+} f(x) =$
- c). $\lim_{x \rightarrow 2} f(x) =$
- d) $f(2) =$
- e). Is $f(x)$ continuous at $x = 2$? Why or why not

7. Given $f(x) = \begin{cases} x + 5 & x < 1 \\ x + 7 & x \geq 1 \end{cases}$

- a). $\lim_{x \rightarrow 1^-} f(x) =$
- b). $\lim_{x \rightarrow 1^+} f(x) =$
- b). $\lim_{x \rightarrow 1} f(x) =$
- d) $f(1) =$
- e). Is $f(x)$ continuous at $x = 1$? Why or why not

Find the indicated limit.

$$8) \lim_{x \rightarrow 2} \frac{x^3 - 2x^2 + 4x - 8}{x^4 - 2x^3 + x - 2}$$

$$9) \lim_{x \rightarrow 2} \frac{x^2 - 4x + 1}{3x - 5}$$

$$10) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

$$11) \lim_{x \rightarrow 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$$

$$12) \lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

$$13) \lim_{x \rightarrow 3} \frac{x^2 - x - 2}{x^2 - 1}$$

$$14) \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x}$$

$$15) \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4} - 4}$$

$$16) \lim_{x \rightarrow 1} \frac{\sqrt{37-x} - 6}{x - 1}$$

$$17) \lim_{t \rightarrow -\infty} \frac{t^3 + 1}{t^3 + 3t^2 + 5}$$

$$18) \lim_{x \rightarrow \infty} \frac{3x - 2}{5x^2 + 4x + 1}$$

$$19) \lim_{x \rightarrow \infty} \frac{2x^2 + 1}{3x - 5}$$

Use the **definition of continuity** to determine whether $f(x)$ is continuous at a .

$$20) f(x) = \frac{x-2}{x^2-4} \quad a = 2$$

$$21) f(x) = \frac{x^2-4}{x-2} \quad a = 3$$

$$22) f(x) = \begin{cases} 5x & \text{if } x < 4 \\ 21 & \text{if } x = 4, \\ x^2 + 4 & \text{if } x > 4 \end{cases} \quad a = 4$$

$$23) f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2, \\ 7 - x & \text{if } x > 2 \end{cases} \quad a = 2$$

Determine for what numbers, if any, the given function is discontinuous. Use the **definition of continuity** to show the discontinuity.

$$24) f(x) = \frac{x+1}{x^2-3x-4}$$

$$25) f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

A piecewise function is given. Use the **properties of limits** to find the indicated limits, or state that a limit does not exist.

$$26) \quad f(x) = \begin{cases} 1-x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ x^2 + 3 & \text{if } x > 1 \end{cases} \quad \text{a) } \lim_{x \rightarrow 1^-} f(x) \quad \text{b) } \lim_{x \rightarrow 1^+} f(x) \quad \text{c) } \lim_{x \rightarrow 1} f(x)$$

$$27) \quad f(x) = \begin{cases} \frac{x^2 - 25}{x + 5} & \text{if } x \neq 5 \\ 13 & \text{if } x = 5 \end{cases} \quad \text{a) } \lim_{x \rightarrow 5^-} f(x) \quad \text{b) } \lim_{x \rightarrow 5^+} f(x) \quad \text{c) } \lim_{x \rightarrow 5} f(x)$$

28) Use the definition of the derivative to find the derivative of $y = 3x^2 - x + 1$.

29) Use the definition of the derivative to find the derivative of $y = \frac{3}{x+1}$.

Answers to Unit-6 Test Review

1) DNE

2) 0

3) DNE

4) 1

5) a. $\lim_{x \rightarrow 3^-} f(x) = \underline{4}$

g. $\lim_{x \rightarrow 1^-} f(x) = \underline{3}$

b. $\lim_{x \rightarrow 3^+} f(x) = \underline{4}$

h. $\lim_{x \rightarrow -1^+} f(x) = \underline{2}$

c. $\lim_{x \rightarrow 3} f(x) = \underline{4}$

i. $\lim_{x \rightarrow -1} f(x) = \underline{\text{DNE}}$

d. $\lim_{x \rightarrow -3^-} f(x) = \underline{-3}$

j. $f(-1) = \underline{-2}$

e. $\lim_{x \rightarrow -3^+} f(x) = \underline{-3}$

k. $f(3) = \underline{\text{undefined}}$

f. $\lim_{x \rightarrow -3} f(x) = \underline{-3}$

6) a). $\lim_{x \rightarrow 2} f(x) = 10$

7) a). $\lim_{x \rightarrow 1^-} f(x) = 6$

b). $\lim_{x \rightarrow 2^+} f(x) = 10$

b). $\lim_{x \rightarrow 1^+} f(x) = 8$

c). $\lim_{x \rightarrow 2} f(x) = 10$

c). $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

d) $f(2) = 10$

d) $f(1) = 8$

e). Is $f(x)$ continuous at $x = 2$? Why or why not

e). Is $f(x)$ continuous at $x = 1$? Why or why not

Yes because $\lim_{x \rightarrow 2} f(x) = f(2) = 10$

No because $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

8) $\frac{8}{9}$

9) -3

10) 2

11) $-\frac{1}{25}$

12) $-\frac{1}{9}$

13) $\frac{1}{2}$

14) $\frac{1}{6}$

15) 0

16) $-\frac{1}{12}$

17) 1

18) 0

19) ∞

20) $f(x)$ is discontinuous at $x = 2$

21) $f(x)$ is continuous at $x = 3$

22) $f(x)$ is discontinuous at $x = 4$

23) $f(x)$ is continuous at $x = 2$

24) $f(x)$ is discontinuous at $x = -1, 4$

25) $f(x)$ is continuous at all values of x

26) a) 0 b) 4 c) DNE

27) a) 0 b) 0 c) 0

28) $y' = 6x - 1$

29) $y' = -\frac{3}{(x+1)^2}$