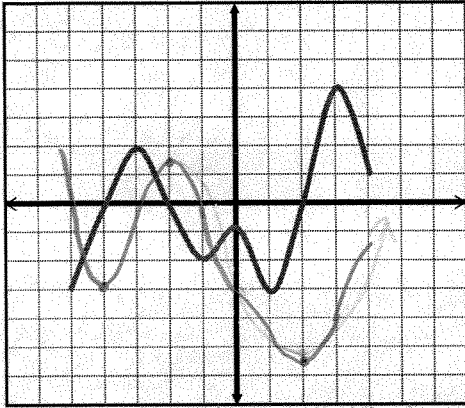


**HW59: Unit 8 Test Review**

1) Giving the graph of the derivative of the function  $f(x)$  below.



a) Sketch the graph of  $f(x)$  on the same graph as the derivative.

b) For what values of  $x$  is  $f(x)$  decreasing?  $(-5, -4) \cup (-2, 2)$

c) For what values of  $x$  does  $f(x)$  have a relative min.?  $-4, 2$

d) For what values of  $x$  is  $f(x)$  concave down?  $(-3, -1) \cup (0, 1) \cup (3, 4)$

e) For what values of  $x$  are inflection points?  $-3, -1, 0, 1, 3$

Find the absolute maximum and minimum of:

2)  $f(x) = 4 - x^2$  on  $[-3, 1]$

$0 = -2x$   
 $x = 0$

$x$	$-3$	$0$	$1$
$y$	$-5$	$4$	$3$

$(0, 4)$  Abs. Max.  
 $(-3, -5)$  Abs. Min.

3)  $y = x^4 - 3x^2 + 4$  on  $[-2, 1]$

$0 = 4x^3 - 6x$   
 $0 = 2x(2x^2 - 3)$   
 $x = 0, \pm 1.2$

$x$	$-2$	$-1.2$	$0$	$1$
$y$	$8$	$1.8$	$4$	$2$

$(-2, 8)$  Abs. Max.  
 $(-1.2, 1.8)$  Abs. Min.

Find the following from the given derivatives:

4)  $y' = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3, -2$

$y'' = 2x - 1$

$x = 1/2$

$f''$   $-$   $-$   $+$   $+$

$f'$   $+$   $-$   $-$   $+$

$x$	$-2$	$1/2$	$3$
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Increasing:  $(-\infty, -2) \cup (3, \infty)$

Decreasing:  $(-2, 3)$

Critical Points:  $x = -2, 3$

Concave Up:  $(1/2, \infty)$

Concave Down:  $(-\infty, 1/2)$

Inflection Points:  $x = 1/2$   
values

5)  $y' = 2x - 6$

$x = 3$

$y'' = 2$

$x = \text{none}$

$f''$   $+$   $+$

$f'$   $-$   $+$

$x$	$3$
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Increasing:  $(3, \infty)$

Decreasing:  $(-\infty, 3)$

Critical Points:  $x = 3$

Concave Up:  $(-\infty, \infty)$

Concave Down: None

Inflection Points: None  
values

Identify the features of and sketch the graph on the given interval.

6)  $f(x) = 2x^3 - 6x + 5$   $-2 \leq x \leq 2$

$f' = 6x^2 - 6$

$x = \pm 1$

$f'' = 12x$

$x = 0$

Increasing:  $(-2, -1) \cup (1, 2)$

Decreasing:  $(-1, 1)$

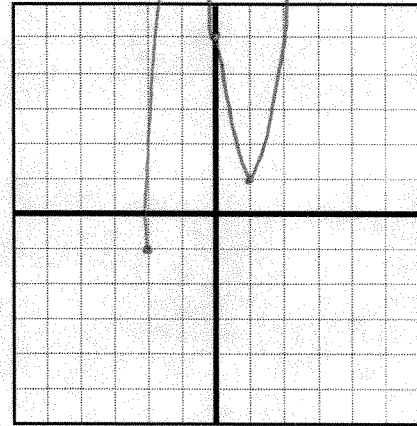
Critical Points:  $(-1, 9), (1, 1)$

Concave Up:  $(0, 2)$

Concave Down:  $(-2, 0)$

Inflection Points:  $(0, 5)$

$f''$	-	-	+	+	
$f'$	+	-	-	+	
$x$	-1	0	1	-2	2
$y$	9	5		-3	9



Find the value of  $c$  that satisfies the Mean Value Theorem.

7)  $f(x) = x^2 - 5x + 4$  on  $[0, 2]$

$(0, 4)$   $(2, -2)$   
 $m = \frac{-2-4}{2-0} = -3$

$-3 = 2c - 5$

$2 = 2c$

$c = 1$

8)  $f(x) = \sqrt{x-1}$  on  $[1, 5]$

$m = (1, 0)$   $(5, 2) = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}(c-1)^{-1/2}$

$1 = \frac{1}{\sqrt{c-1}}$

$\sqrt{c-1} = 1$

$c-1 = 1$

$c = 2$

Find the critical points of the function. Round to the nearest tenth.

9)  $f(x) = \frac{x^2 - 5x + 4}{x + 2}$

$f' = \frac{(2x-5)(x+2) - (x^2-5x+4)}{(x+2)^2}$

$f' = \frac{2x^2 - x - 10 - x^2 + 5x - 4}{(x+2)^2}$

$f' = \frac{x^2 + 4x - 14}{(x+2)^2}$

$0 = x^2 + 4x - 14$

$x^2 + 4x + 4 = 18$

$(x+2)^2 = 18$

$x = 2.2, -6.2$

$(2.2, -0.5)$   $(-6.2, -17.5)$

10)  $f(x) = 4x^{1/3} - x^{4/3}$

$f' = \frac{4}{3}x^{-2/3} - \frac{4}{3}x^{1/3}$

$f' = \frac{4}{3}x^{-2/3}(1-x)$

$x = 0, 1$

$(1, 3)$

$(0, 0)$  does not change direction

Find the inflection points of the function. Round to the nearest tenth.

11)  $f(x) = \frac{1}{2}x^4 - 3x^3 + 2$       $\frac{81}{2} - \frac{162}{2} + \frac{4}{2}$

$f' = 2x^3 - 9x^2$

$f'' = 6x^2 - 18x$

$0 = 6x(x-3)$

$x = 0, 3$

$(0, 2)$   
 $(3, -\frac{77}{2})$

$f''$	+	-	+
$x$	0	3	
$y$	2	$-\frac{77}{2}$	

12)  $f(x) = x^4 - 3x^3 + x + 4$

$f' = 4x^3 - 9x^2 + 1$

$f'' = 12x^2 - 18x$

$f'' = 6x(2x-3)$

$x = 0, \frac{3}{2}$

$(0, 4)$   
 $(\frac{3}{2}, \frac{7}{16})$

$f''$			
$x$	0	1.5	
$y$	4	$\frac{7}{16}$	

Sketch the graph with the given table.

13)

$x$	$-4 \leq x < -1$	$-1$	$-1 < x < 1$	$1$	$1 < x < 3$	$3$	$3 < x \leq 5$
$f(x)$	Negative	-2	Negative	-5	Negative	0	Negative
$f'(x)$	Negative	Zero	Negative	Zero	Positive	Undefined	Negative
$f''(x)$	Negative	Zero	Positive	Zero	Positive	Undefined	Positive

